

A sub-quadratic algorithm to determine if a polyomino tiles the plane by translation

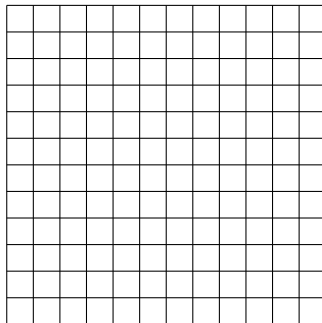
Xavier Provençal



18 mai, 2010

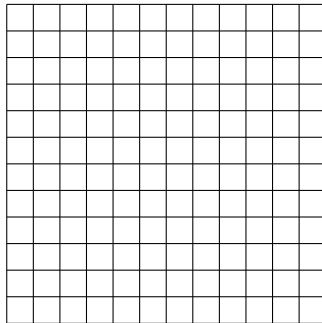
Introduction to polyominos

- Discrete plane : \mathbb{Z}^2



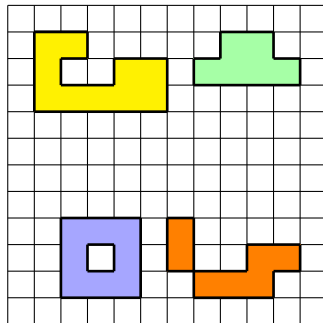
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- Discrete plane : \mathbb{Z}^2
- **Definition** : A *polyomino* is a finite, 4-connected subset of the plane, without holes.



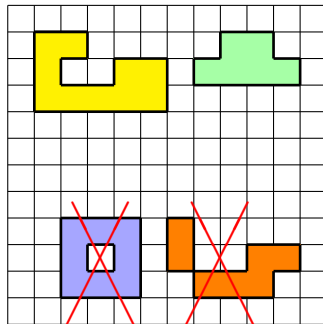
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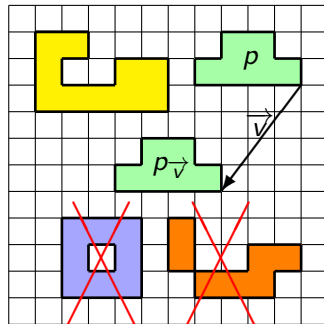
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- Discrete plane : \mathbb{Z}^2
- **Definition** : A *polyomino* is a finite, 4-connected subset of the plane, without holes.
- **Notation** : Let p be a polyomino and \vec{v} a vector of \mathbb{Z}^2 , $p_{\vec{v}}$ will denote the image of p by de translation \vec{v} .



Tiling

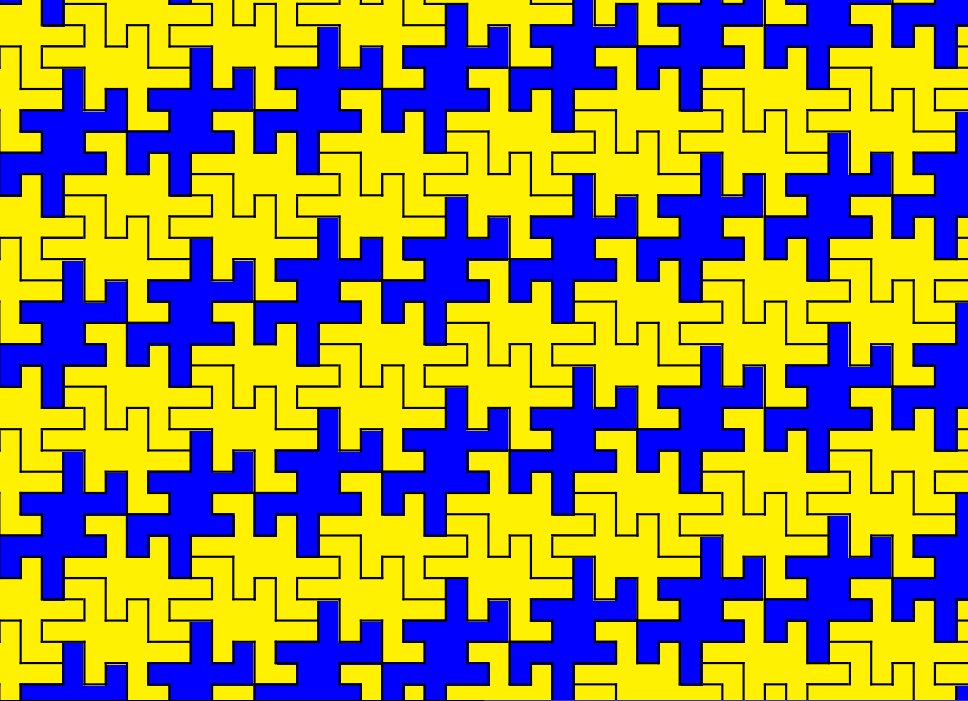
Definition

A tiling of the discrete plane \mathbb{Z}^2 by a set of polyominoes \mathcal{P} is a subset $\mathcal{T} \subset \mathcal{P} \times \mathbb{Z}^2$ such that :

- 1 $\bigcup_{(P, \vec{v}) \in \mathcal{T}} P_{\vec{v}} = \mathbb{Z}^2$
- 2 For any distinct pair $(P, \vec{u}), (Q, \vec{v}) \in \mathcal{T}$, $P_{\vec{u}} \cap Q_{\vec{v}} = \emptyset$.

Example





The tiling problem

Problem (The tiling problem)

Given a set \mathcal{P} , does there exist a tiling of the plane by \mathcal{P} ?

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Theorem (Ollinger, 2008)

The tiling problem with $|\mathcal{P}| = 11$ is undecidable.

Notation

We say that a polyomino P tiles the plane if there exist a tiling of the plane by $\{P\}$.

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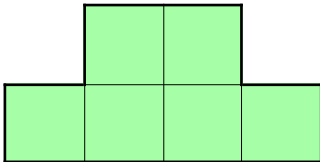
Theorem (Gambini and Vuillon, 2003)

There exist an algorithm in $\mathcal{O}(n^2)$ time that tests if a polyomino tiles the plane.

The Beauquier-Nivat characterization

Freeman chain code

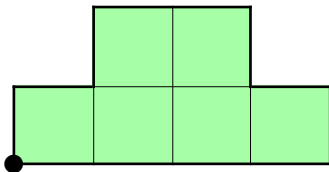
$$\Sigma = \{0, \bar{0}, 1, \bar{1}\}$$



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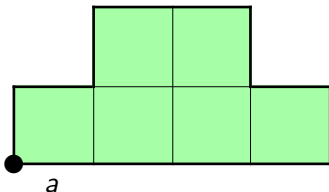


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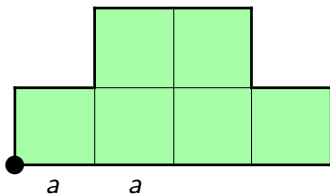


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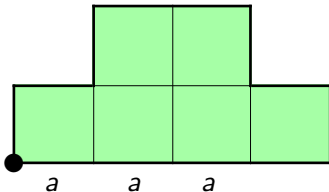


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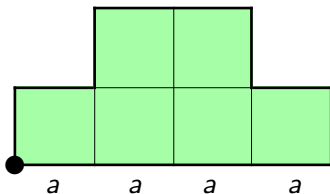


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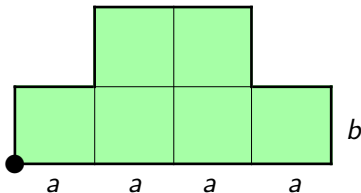


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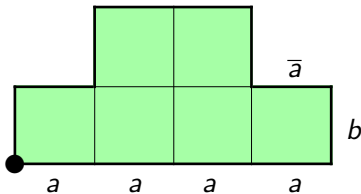


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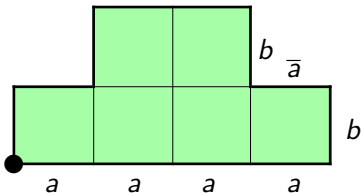


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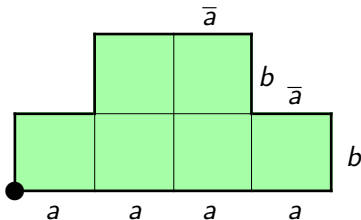


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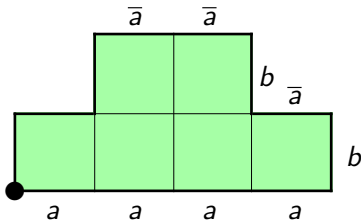


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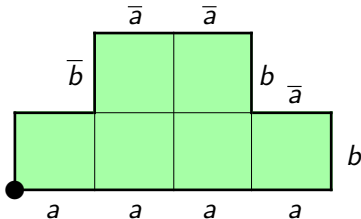


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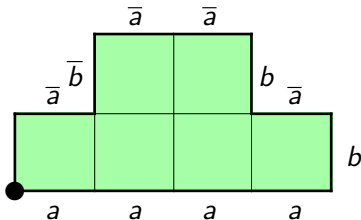


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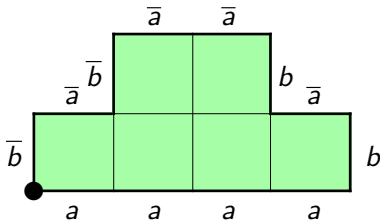


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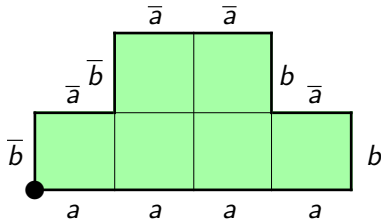
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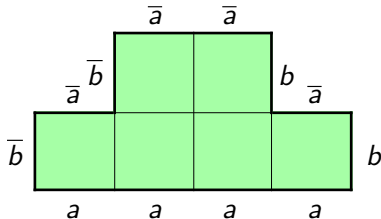
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$$w = uv \text{ and } w' = vu.$$

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Neighbouring

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Two polyominoes p and q are simply neighbouring if

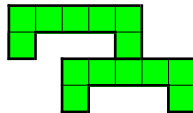
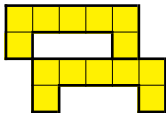
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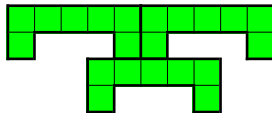
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Surrounding

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A surrounding of the polyomino p is an ordered sequence of translated copies $(p_0, p_1, \dots, p_{k-1})$ such that for every i from 0 to $k - 1$, the polyominoes p , p_i and p_{i+1} form a triad.

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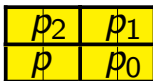
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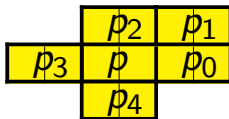
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Surroundings and tilings

Remark

If the polyominoes p and q are simply neighbouring then for any vector \vec{u} the polyominoes $p_{\vec{u}}$ and $q_{\vec{u}}$ are also simply neighbouring.

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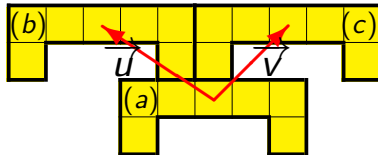
Proposition

A polyomino p tiles the plane if and only if it admits a surrounding.

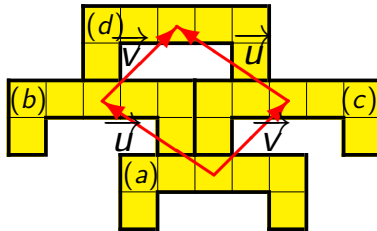
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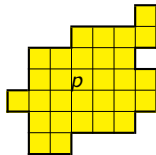
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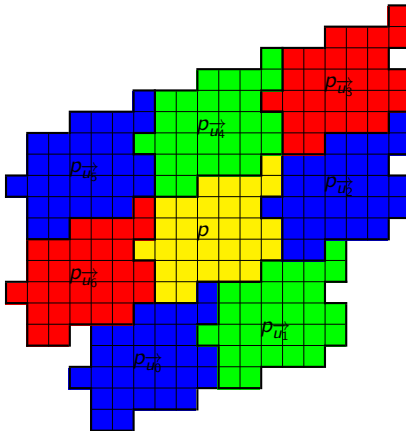
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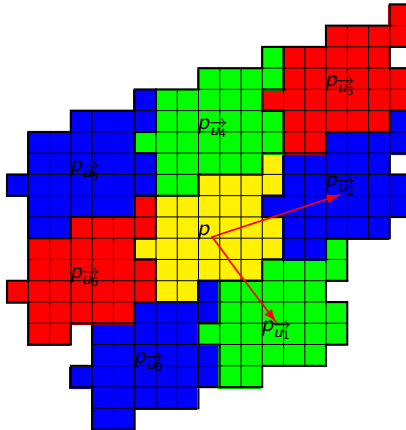
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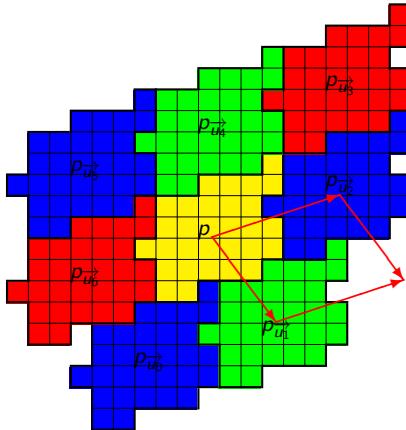
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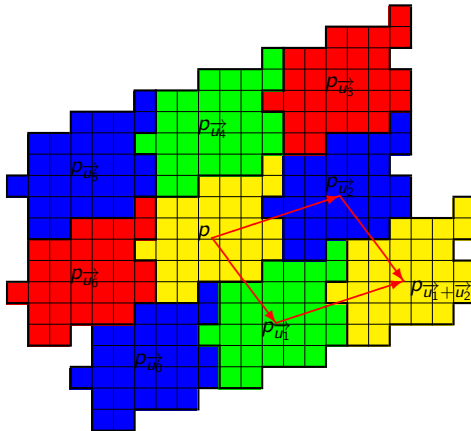
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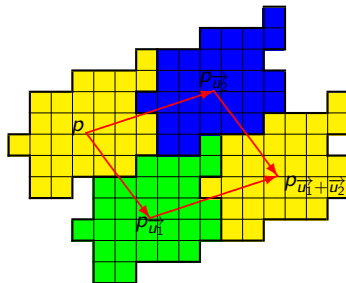
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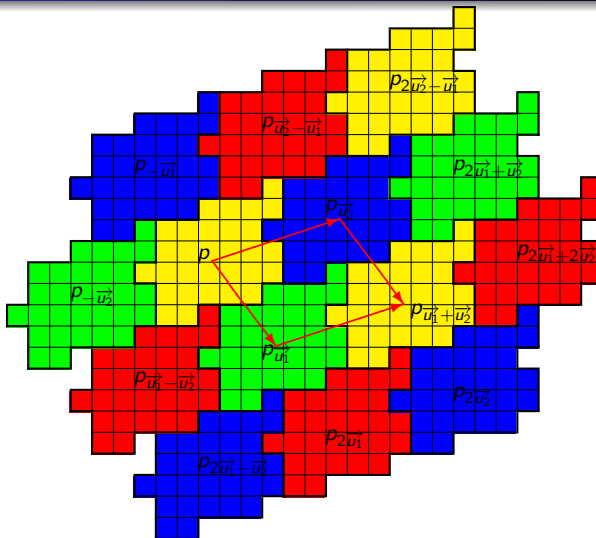
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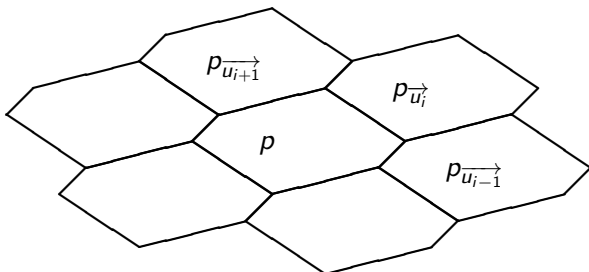


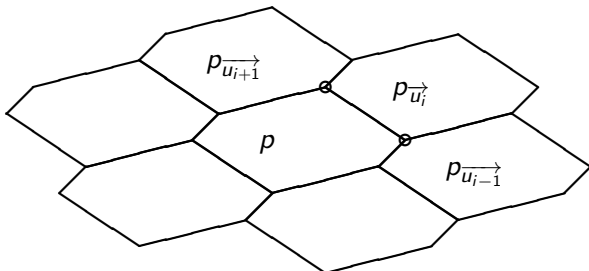
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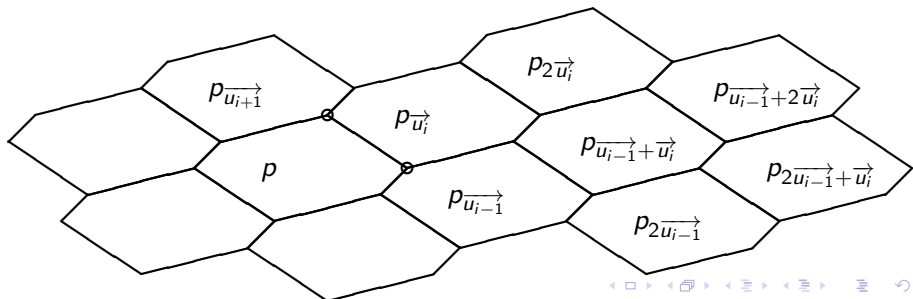


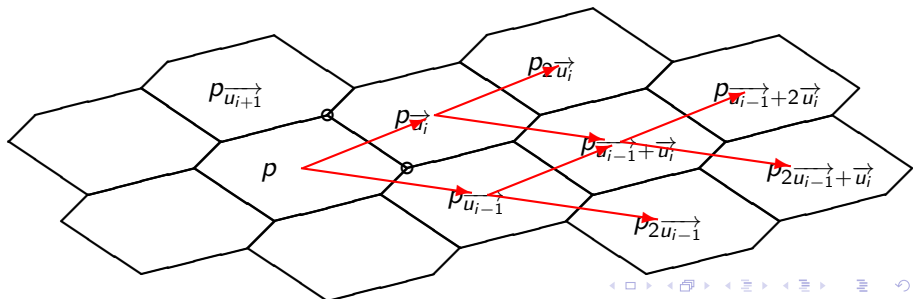
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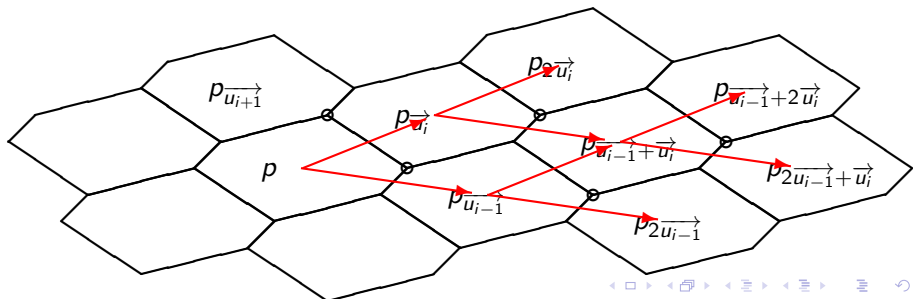


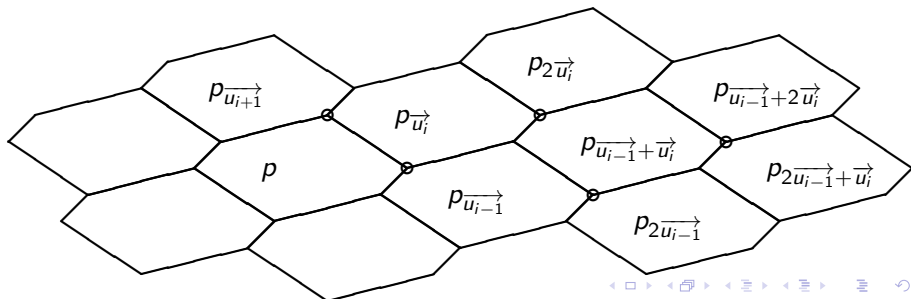


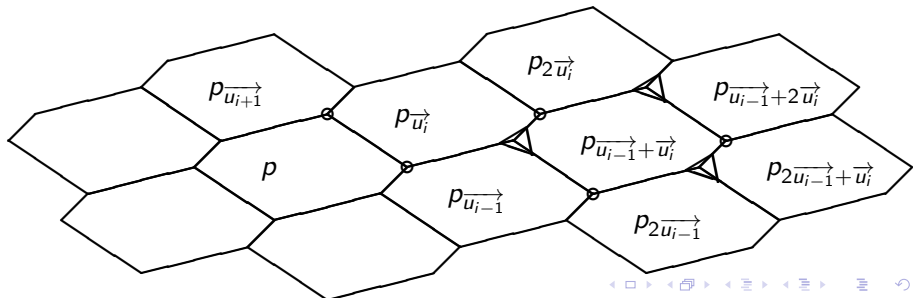


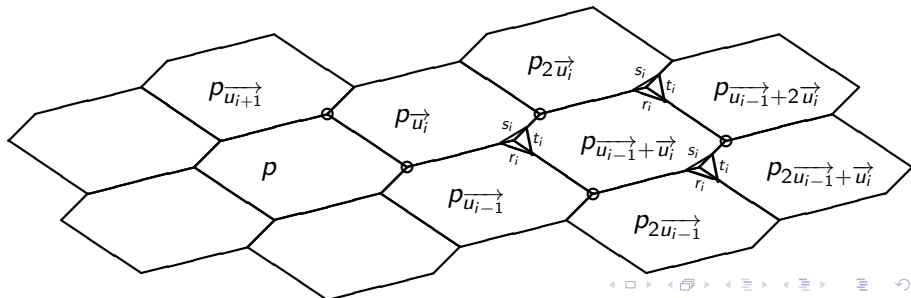


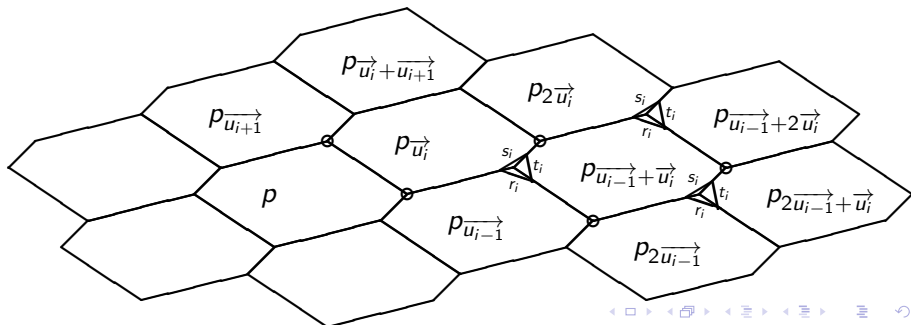


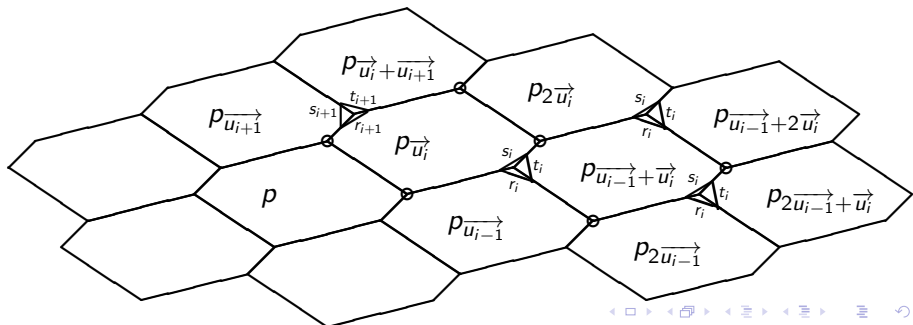


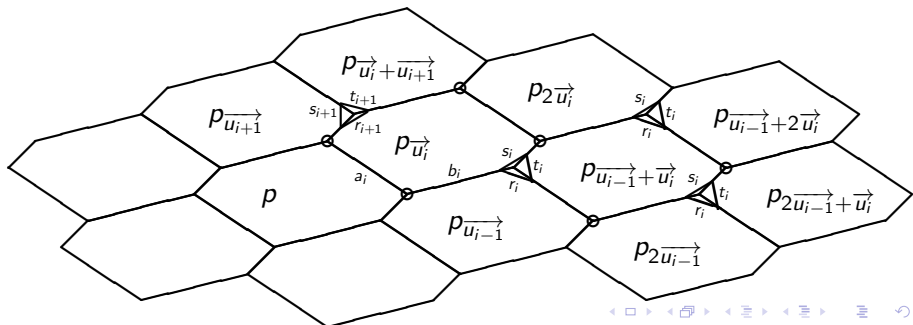




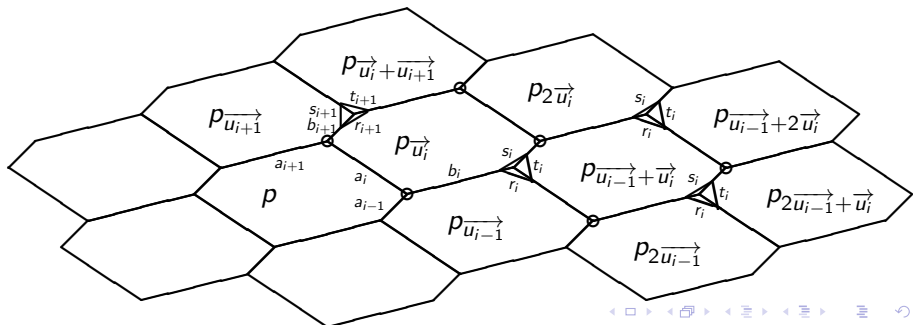






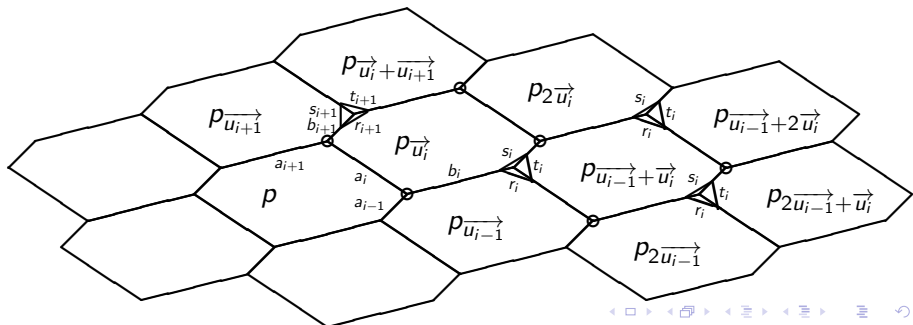


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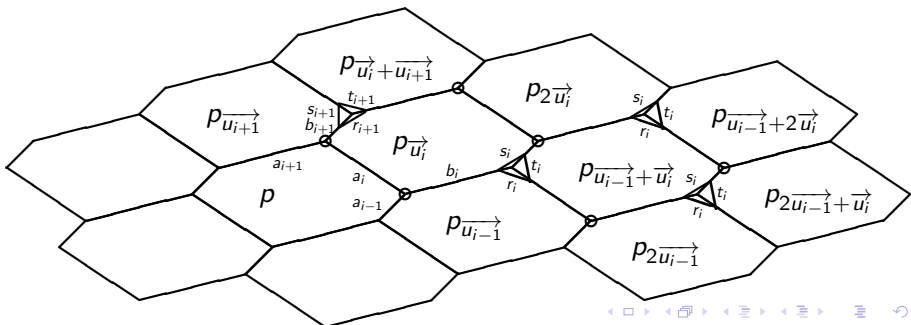
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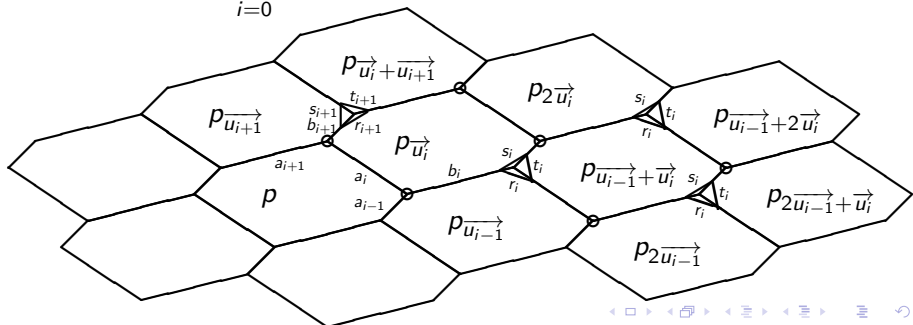


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Surroundings and the factorization

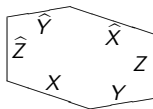
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Surroundings and the factorization

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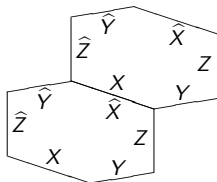
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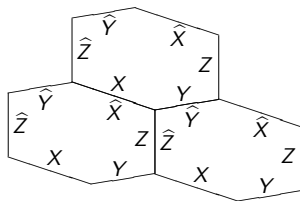
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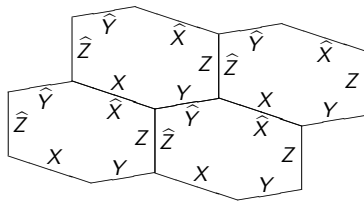
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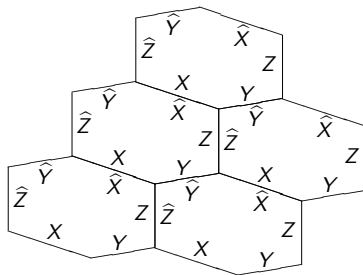
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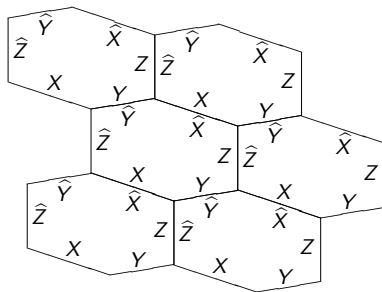
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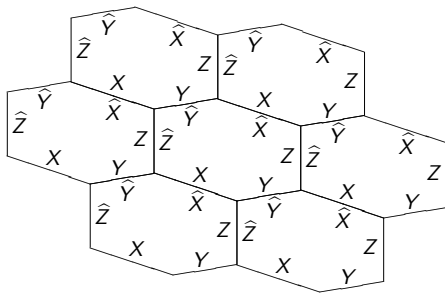
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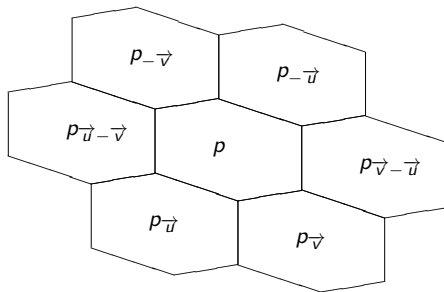
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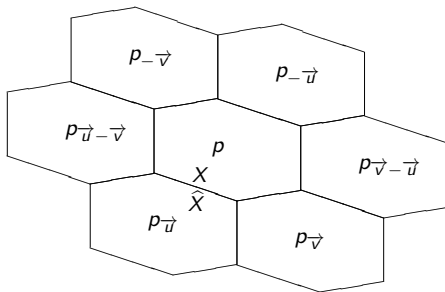
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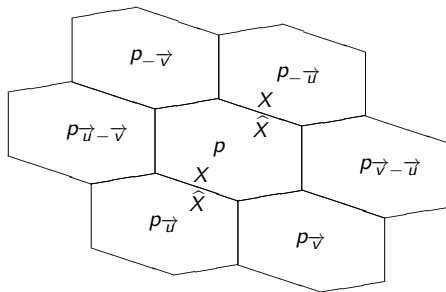
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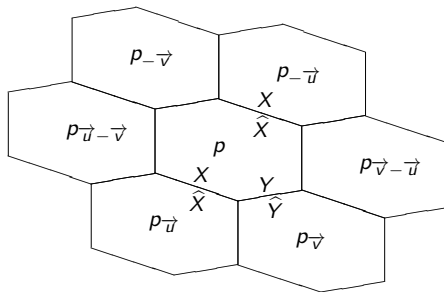
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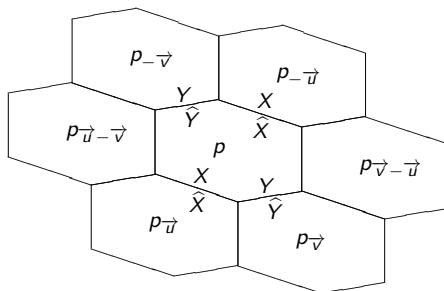
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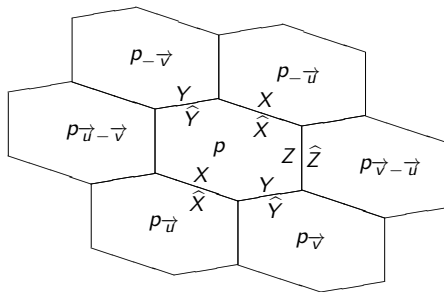
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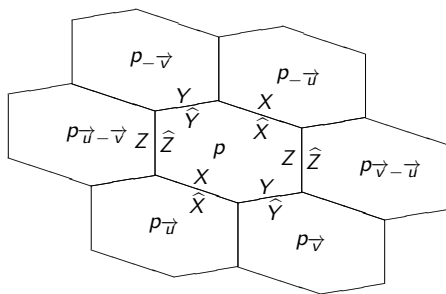
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Pseudo-square and pseudo-hexagons

Definition

A polyomino with Beauquier-Nivat factorization $XYZ\hat{X}\hat{Y}\hat{Z}$ is called a pseudo-square if one of the factors X, Y, Z is the empty word. It is called a pseudo-hexagon otherwise.

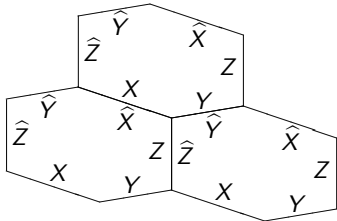
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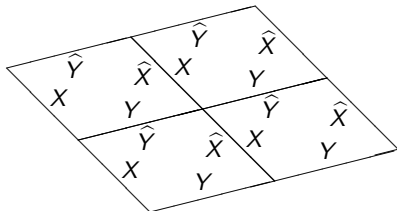
Pseudo-hexagon

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Pseudo-square

$$w \equiv XY\hat{X}\hat{Y}.$$



Admissible factors

Admissible factors

Definition

Given an occurrence of a factor A in the word w coding a polyomino p . This occurrence of the factor A is admissible if

- $w \equiv Ax\hat{A}y$, for x, y such that $|x| = |y|$.
- A is maximal, that is, $\text{first}(x) \neq \overline{\text{last}(x)}$ and $\text{first}(y) \neq \overline{\text{last}(y)}$.

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Notation

Given an admissible occurrence of a factor A in w , the occurrence of the factor \hat{A} such that $w \equiv Ax\hat{A}y$ as in the previous definition is called its homologue.

Admissible factors

Proposition

Let \mathcal{A} be the set of all admissible factors overlapping a position α in w and $\widehat{\mathcal{A}}$ be the set of their respective homologous factors. Then, there is at least one position in w that is not covered by any element of $\mathcal{A} \cup \widehat{\mathcal{A}}$.

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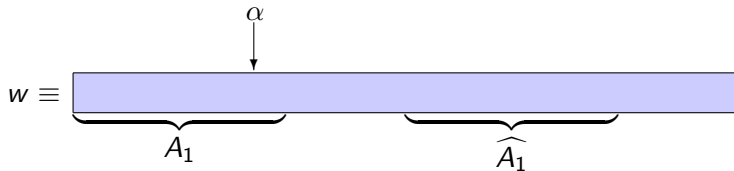
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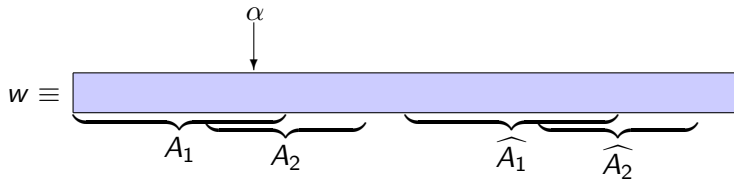
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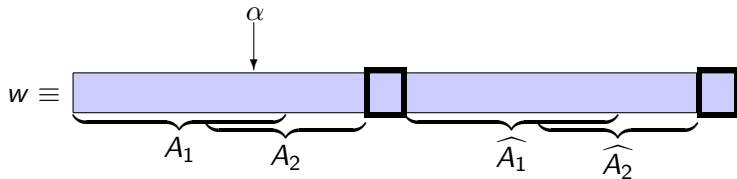
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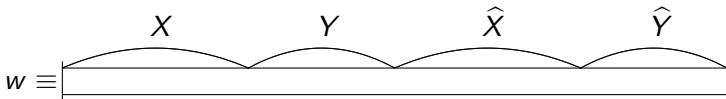
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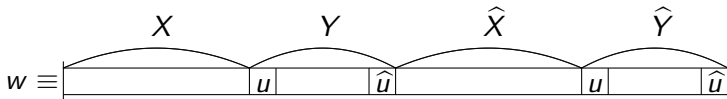
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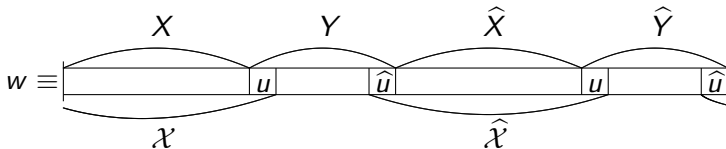
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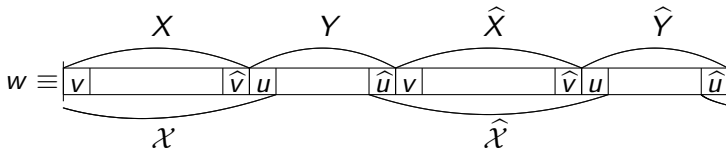
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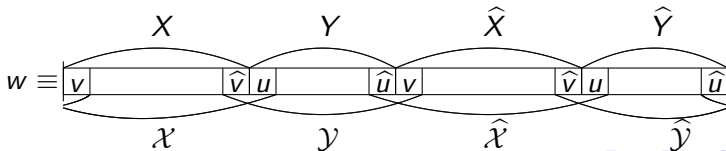
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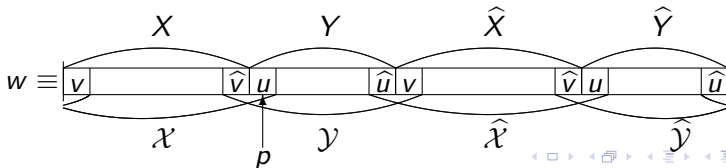
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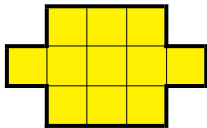
- A is maximal, that is, $\text{first}(x) \neq \overline{\text{last}(x)}$ and $\text{first}(y) \neq \overline{\text{last}(y)}$.

$w \equiv XY\widehat{X}\widehat{Y}$ with $Y = \alpha Y'\overline{\alpha}$.



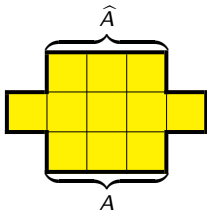
Admissible factors

$$w \equiv a a a b a b \bar{a} b \bar{a} \bar{a} \bar{a} \bar{b} \bar{a} \bar{b} a \bar{b}$$



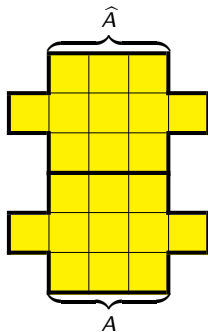
Admissible factors

$$w \equiv \underbrace{a a a}_A \underbrace{b a b \bar{a} b}_x \underbrace{\bar{a} \bar{a} \bar{a}}_{\hat{A}} \underbrace{\bar{b} \bar{a} \bar{b} a \bar{b}}_y$$



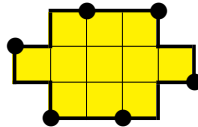
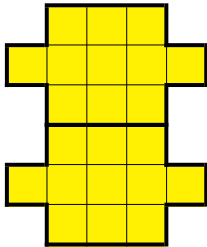
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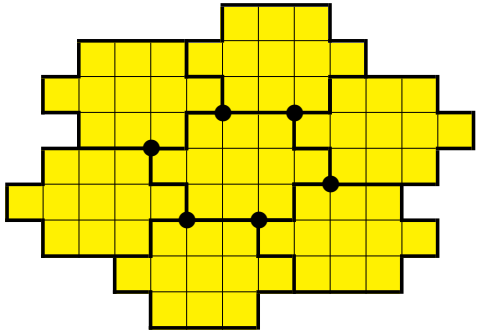
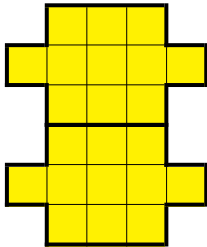
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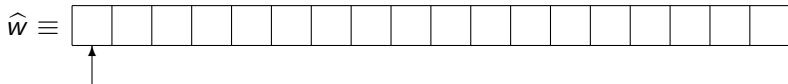
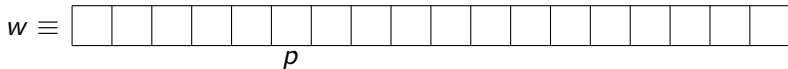
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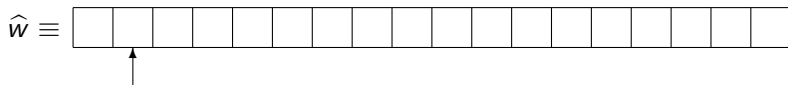
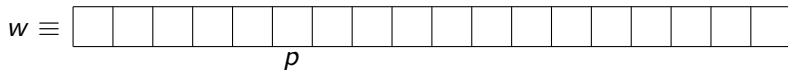


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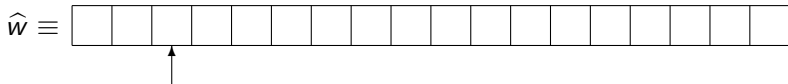
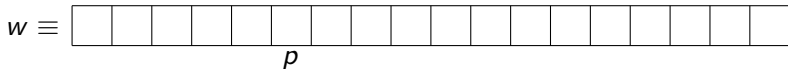


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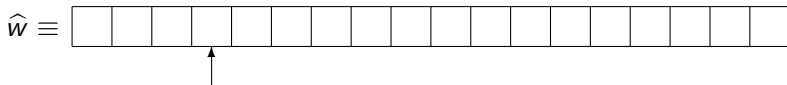
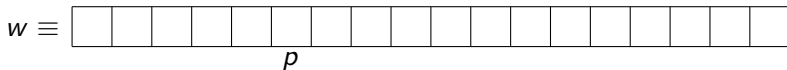


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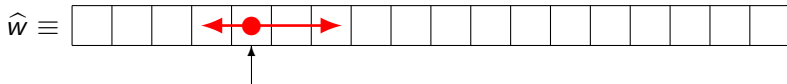
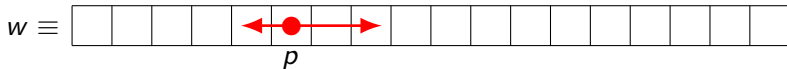


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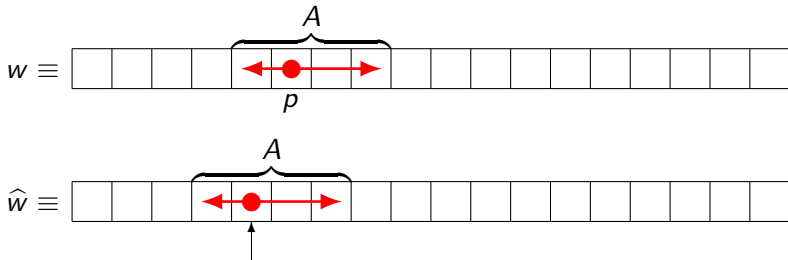


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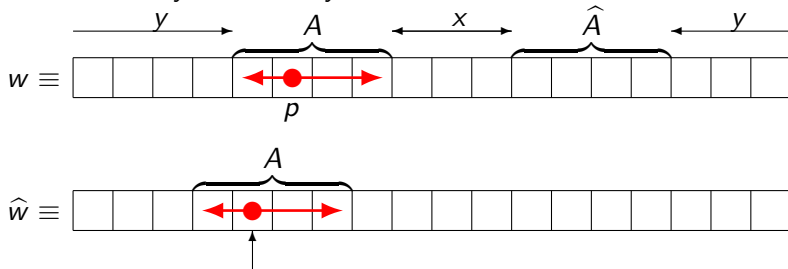


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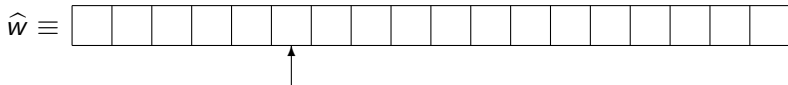
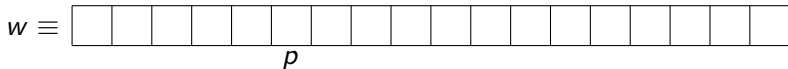


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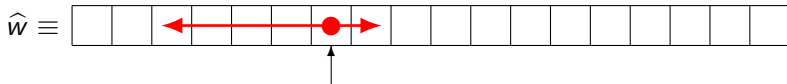
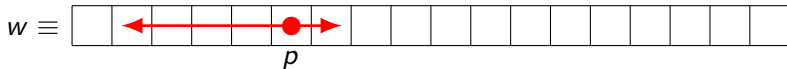


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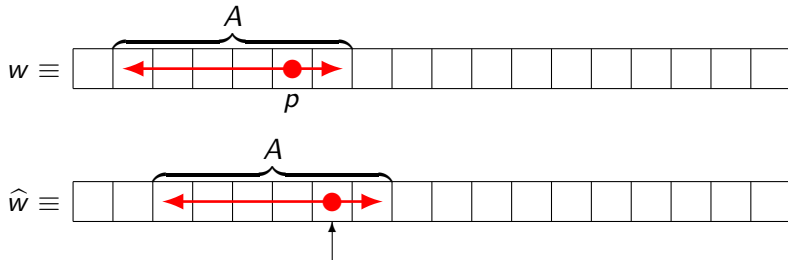


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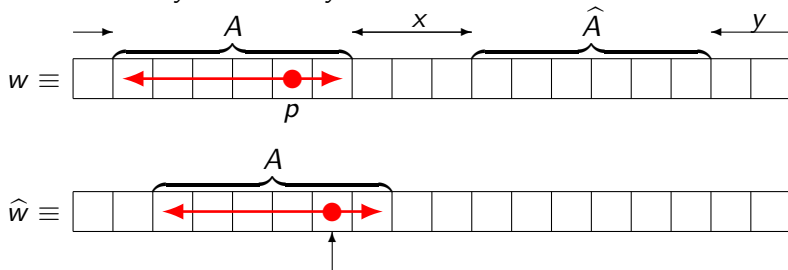


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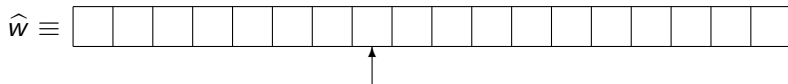
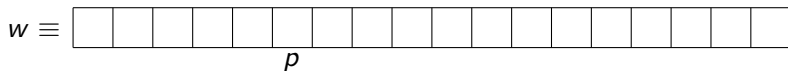


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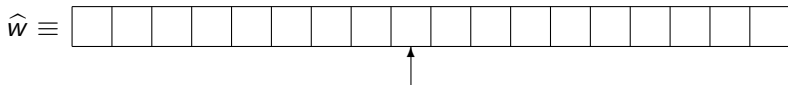
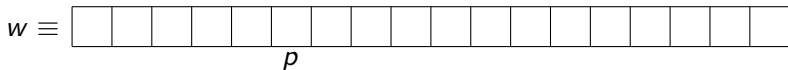


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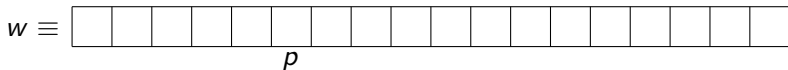


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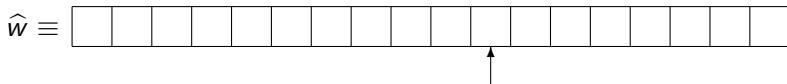
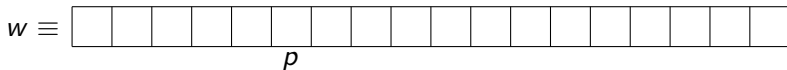


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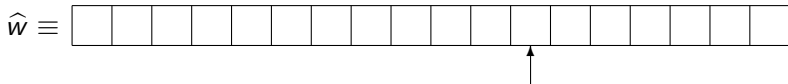
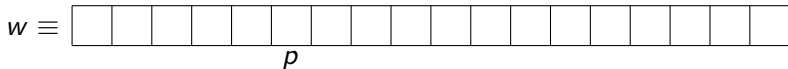


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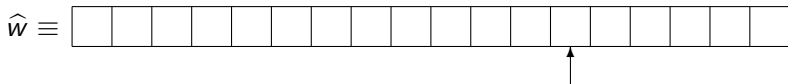
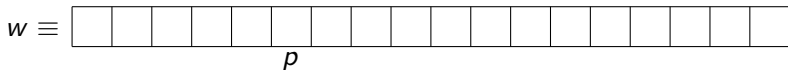


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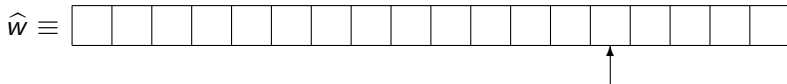
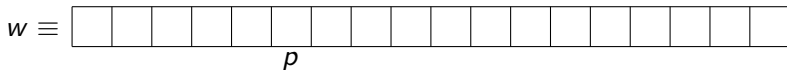


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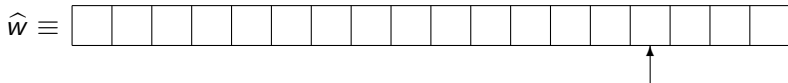
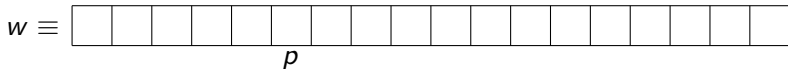


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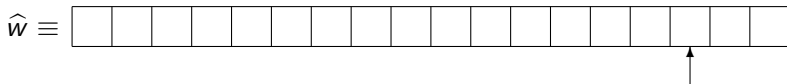
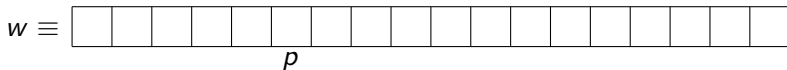


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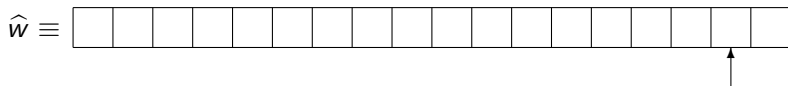
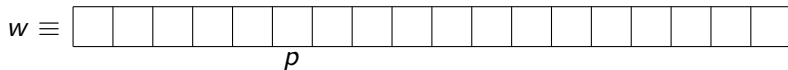


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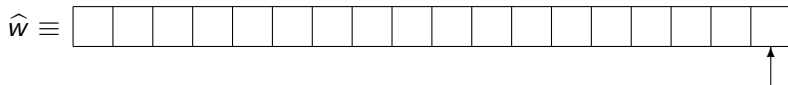
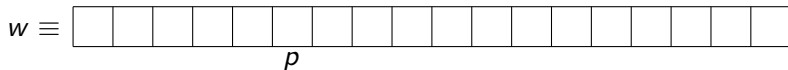


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The algorithms

Detecting pseudo-squares

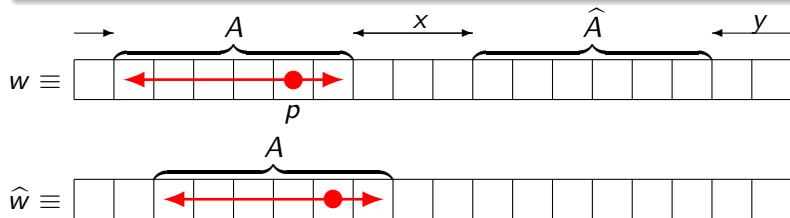
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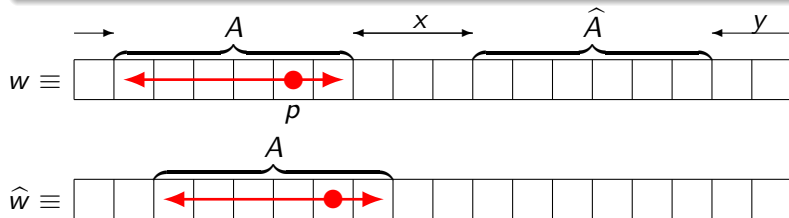
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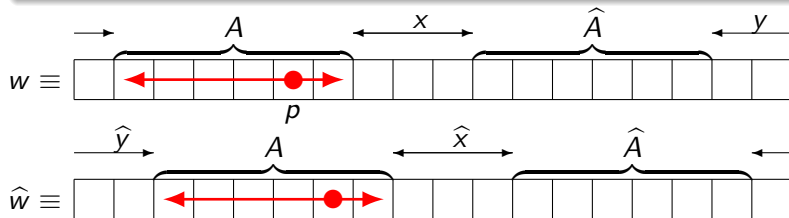


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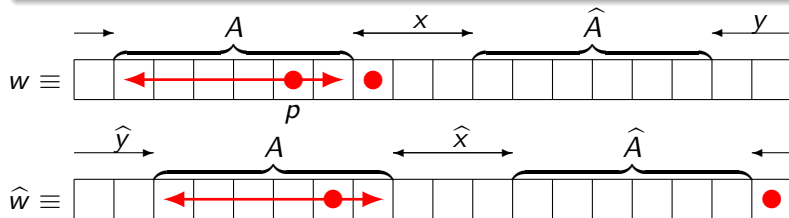
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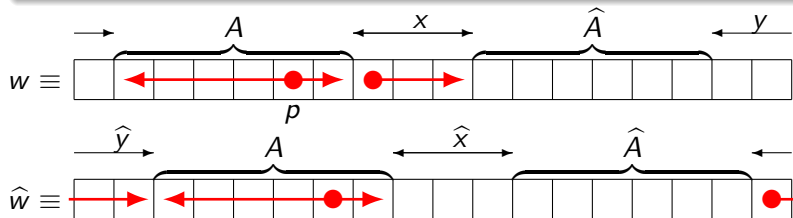
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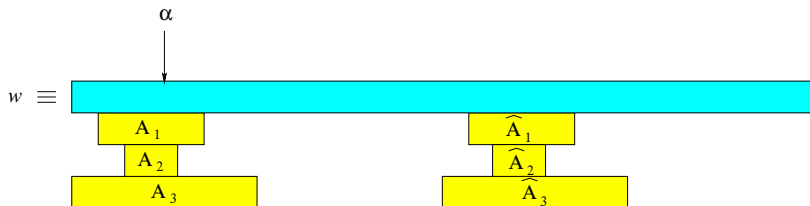
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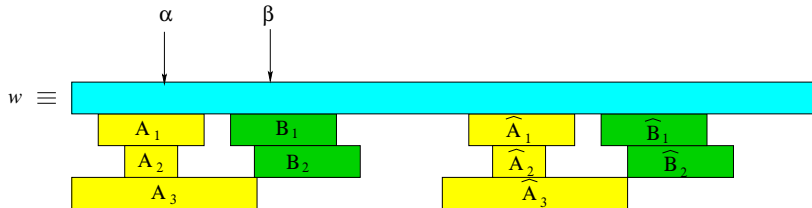
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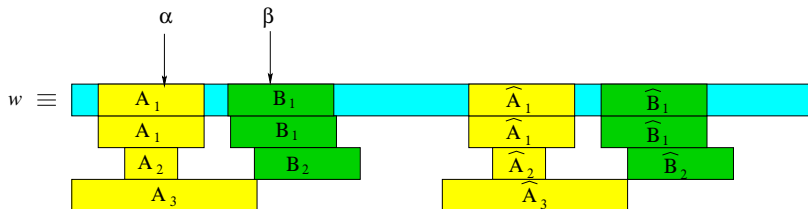
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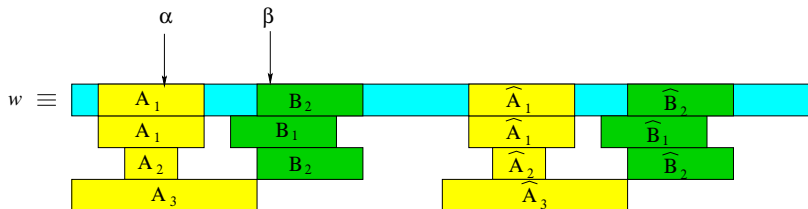
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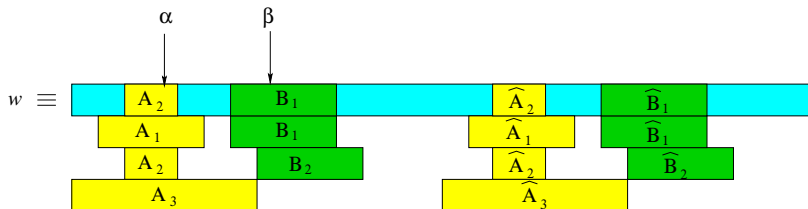
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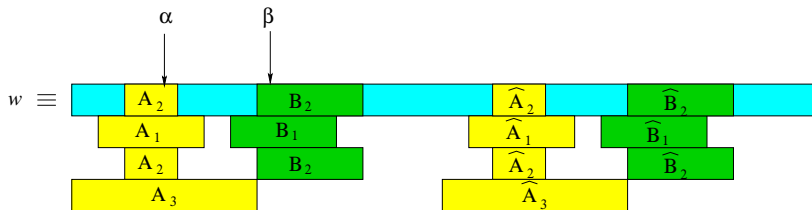
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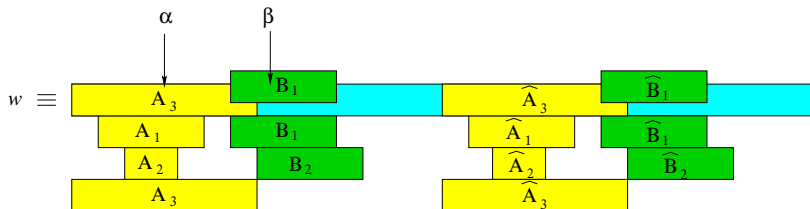
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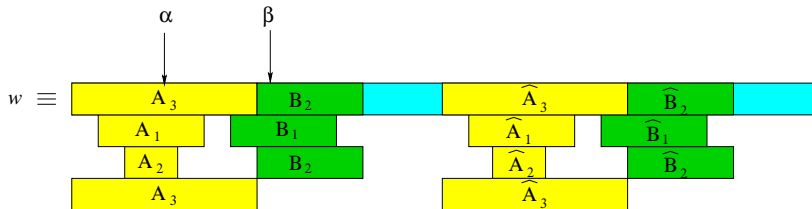
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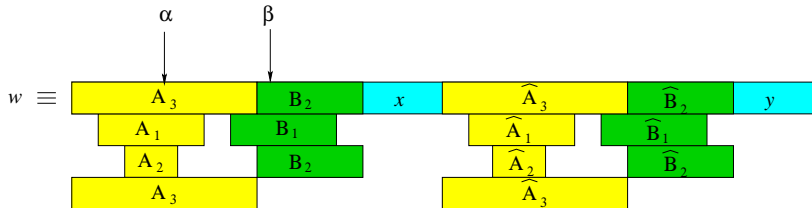
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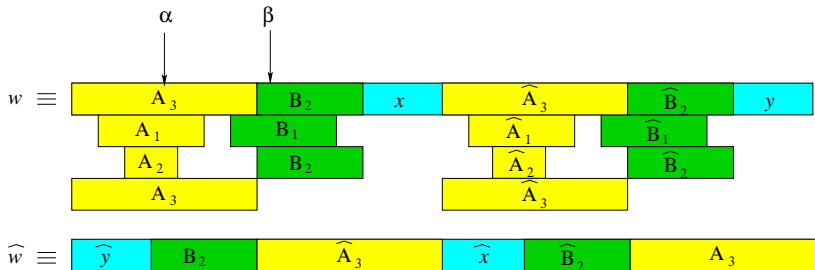
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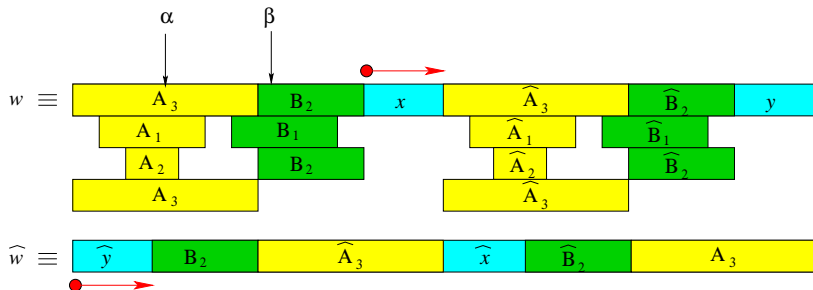
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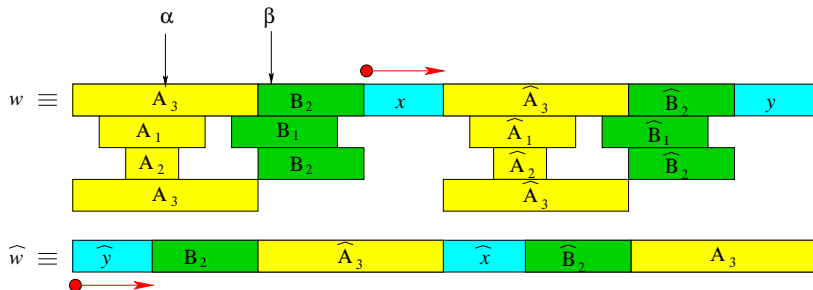
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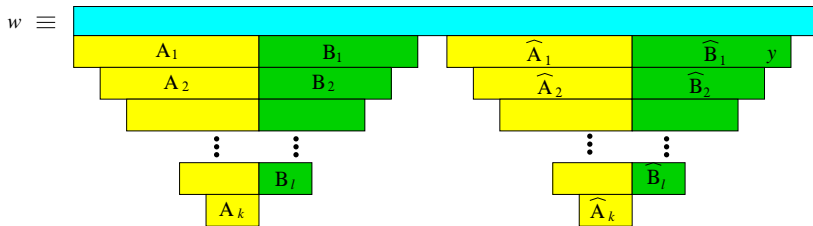
Detecting pseudo-hexagons



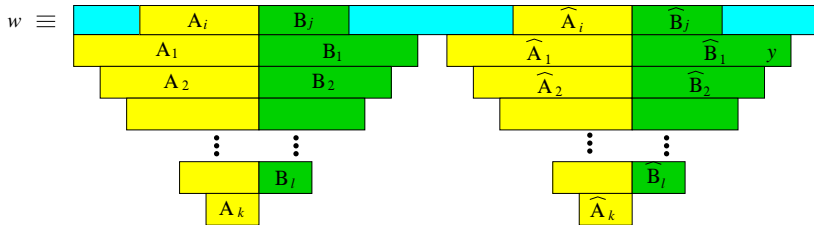
Theorem (Brlek, Fédou and Provençal, 2009)

Given a contour word w of length n such that its longest factor of the form xx is shorter than \sqrt{n} , then this algorithm tests if it is a pseudo-hexagon in $\mathcal{O}(n)$.

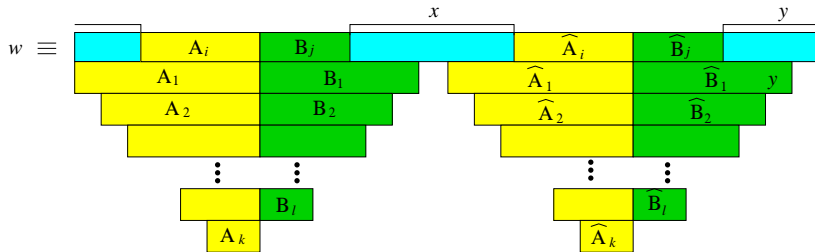
Worst case



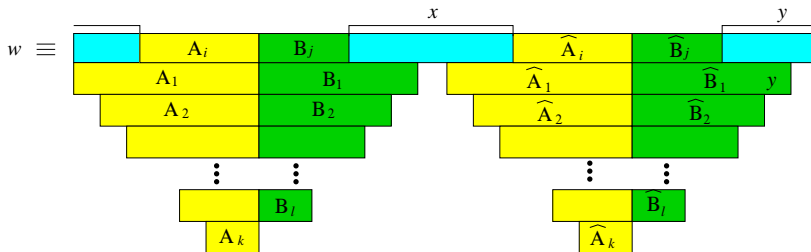
Worst case



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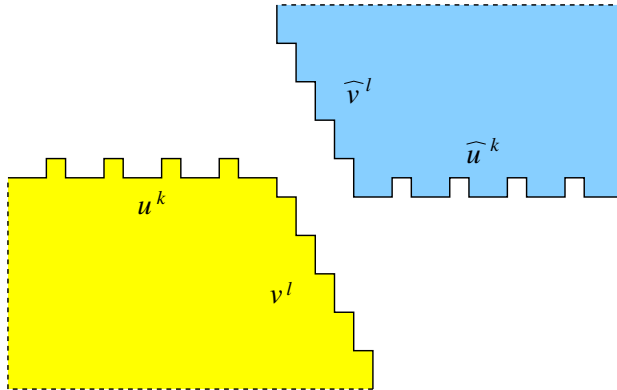


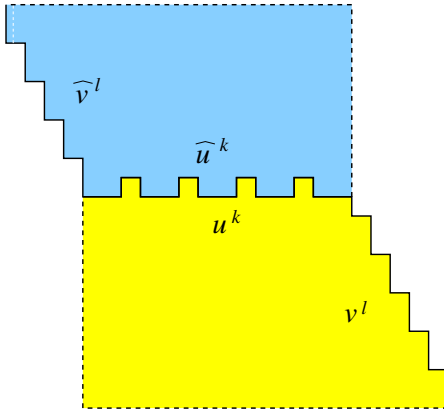
Worst case

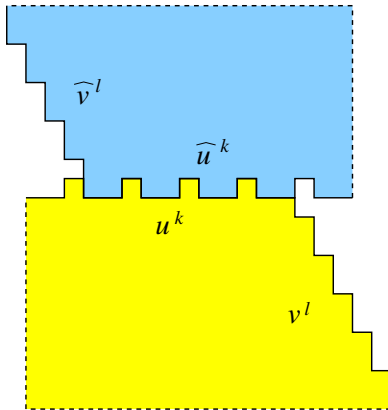


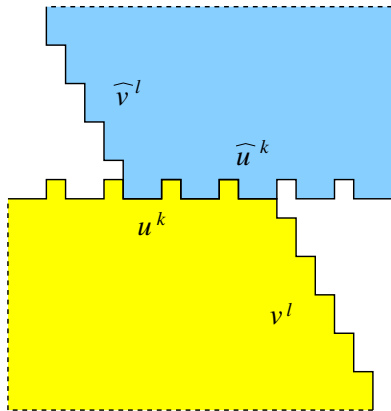
Remark

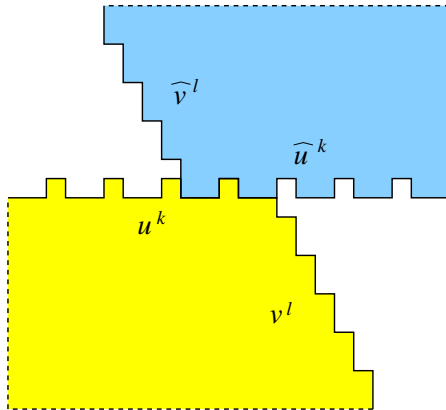
Since k and l can be in $\mathcal{O}(n)$ the number of tests may raise to $\mathcal{O}(n^2)$.

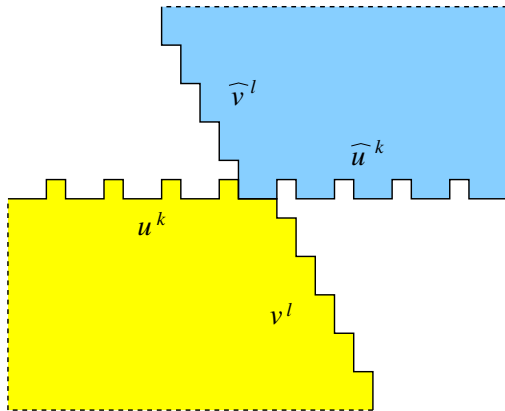


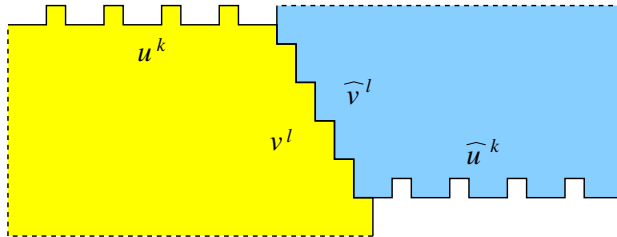


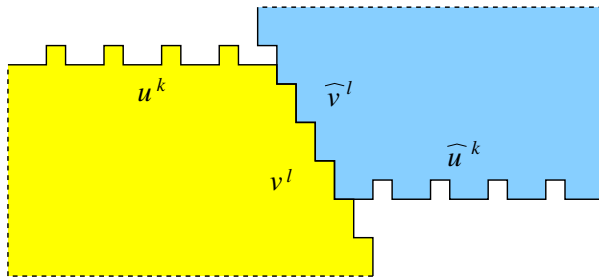


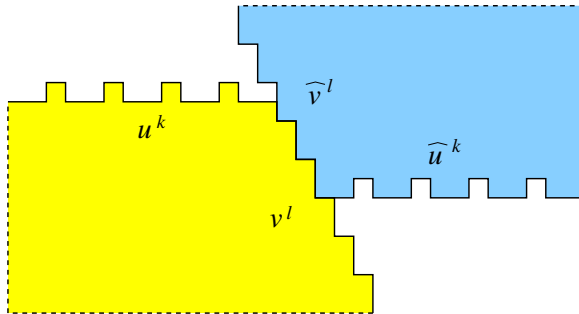


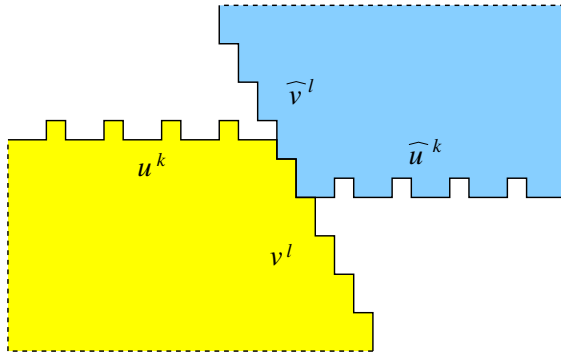


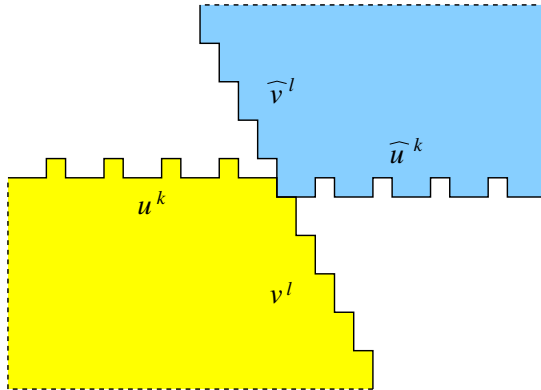


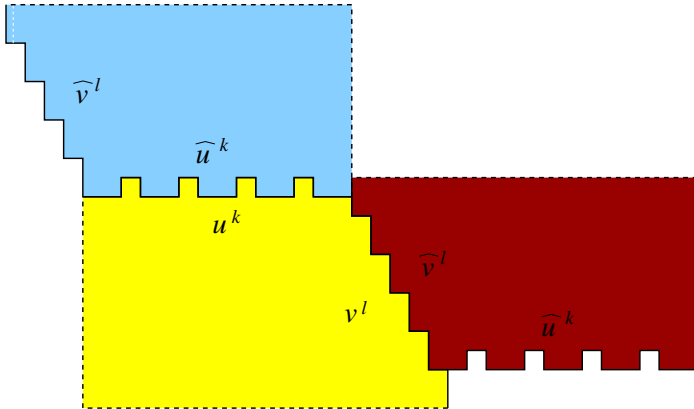


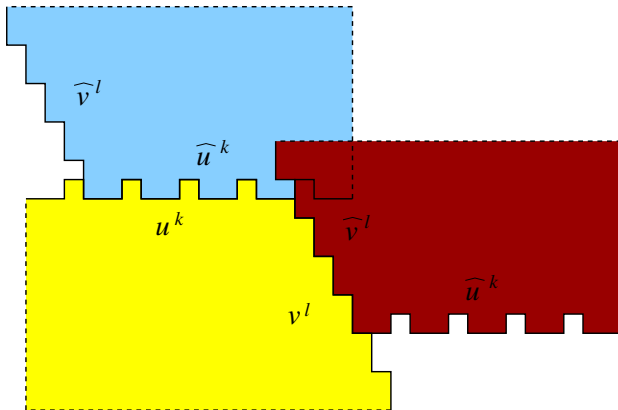


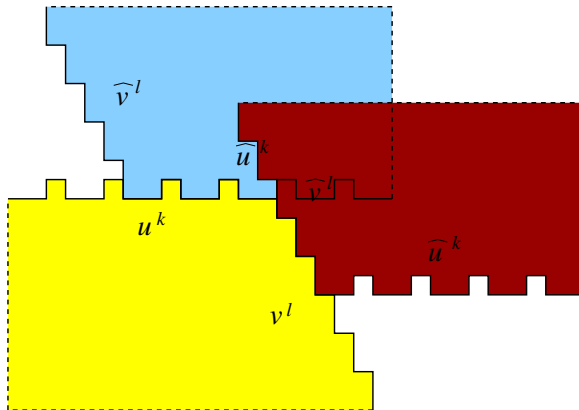


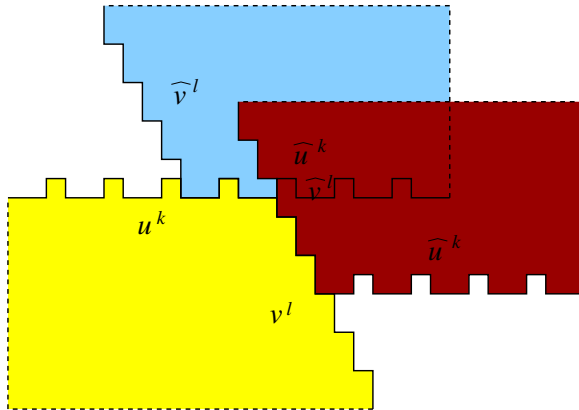












Periodicity in words

Definition

A word $w = w_1 w_2 \cdots w_n$ has a period $p \geq 1$ if $w_i = w_{i+p}$ for all $i \in \{1, 2, \dots, n - p\}$.

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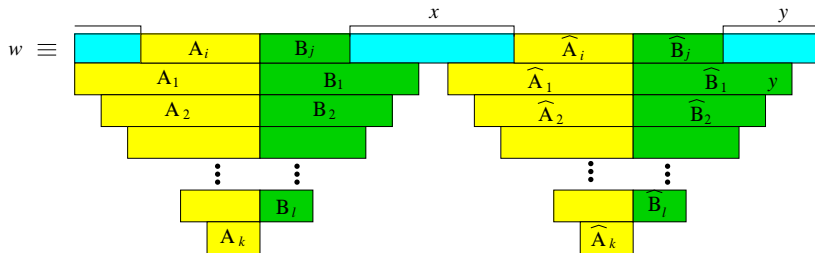
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Theorem (Fine and Wilf, 1965)

If a word w admit two periods p and q such that $|w| \geq p + q - \gcd(p, q)$ then $\gcd(p, q)$ is also a period of w .

Worst case



Theorem (Provençal, 2008)

Testing if a polyomino tiles the plane can be done in $\mathcal{O}(n(\log n)^3)$ time.

MERCI!