On the problem of deciding if a polyomino tiles the plane by translation

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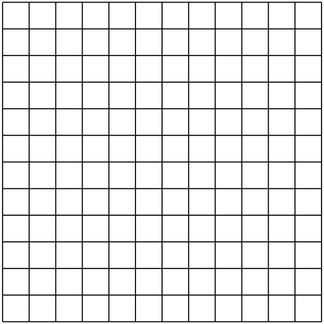
Université du Québec à Montréal,

August 29, 2006

Introduction to polyominos

- Discrete plane : $\mathbb{Z}^2$



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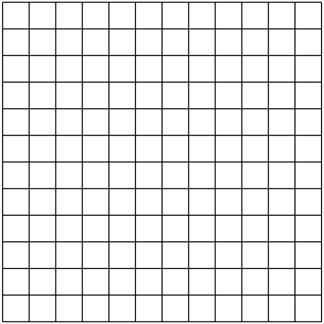

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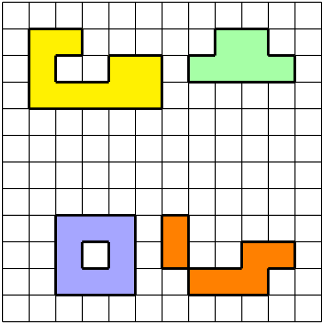

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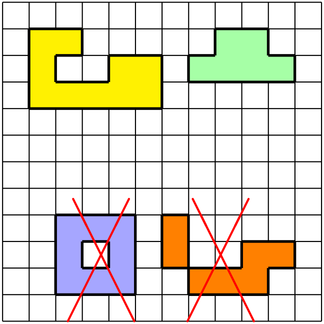

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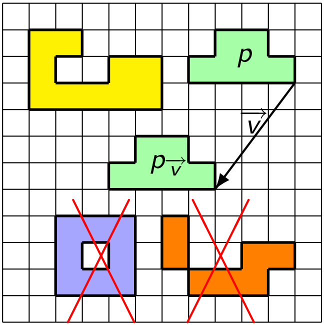
- Discrete plane : $\mathbb{Z}^2$
- **Definition** : A *polyomino* is a finite, 4-connected subset of the plane, without holes.
- **Notation** : Let p be a polyomino and $\overrightarrow{v}$ a vector o $\mathbb{Z}^2$, $p_{\overrightarrow{v}}$ will denote the image of p by de translation $\overrightarrow{v}$.



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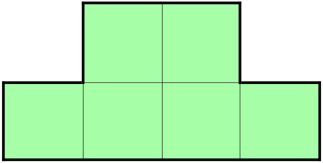
$$\Sigma = \{a, \overline{a}, b, \overline{b}\}$$



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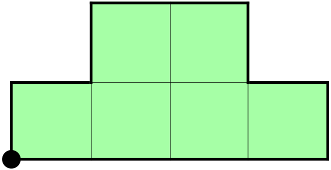

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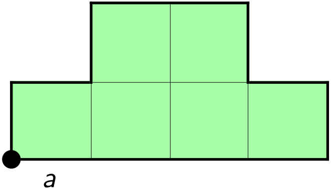

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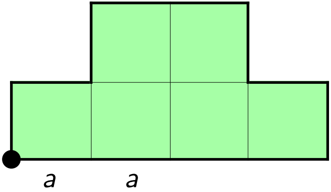

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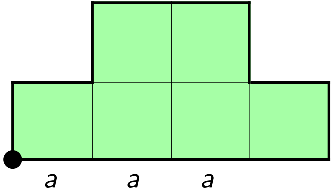

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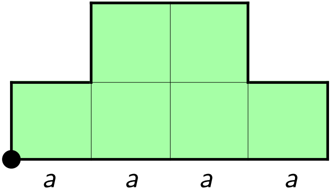

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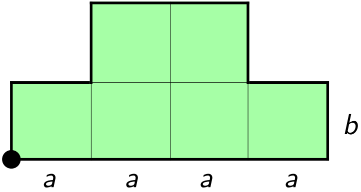

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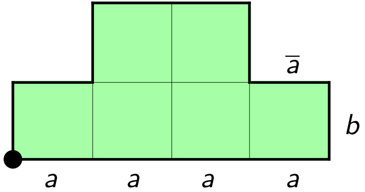

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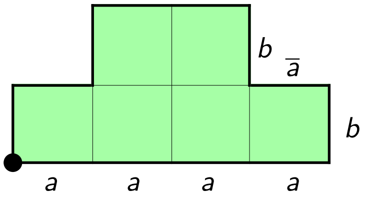

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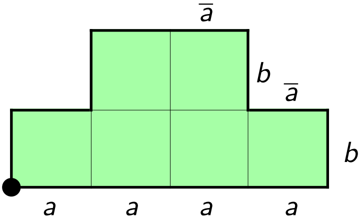

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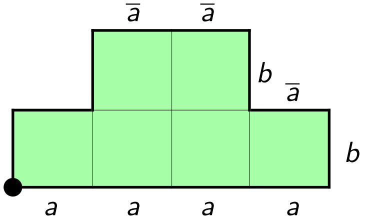

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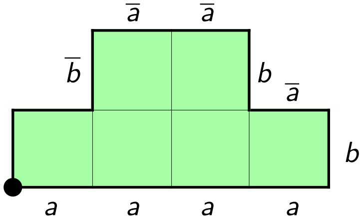

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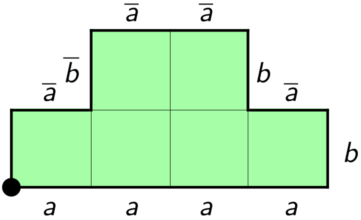

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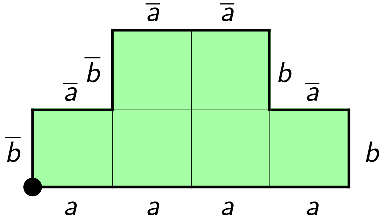

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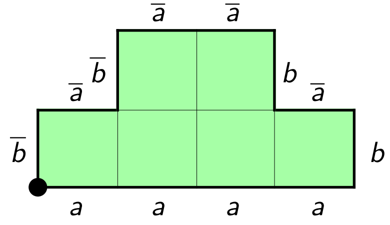

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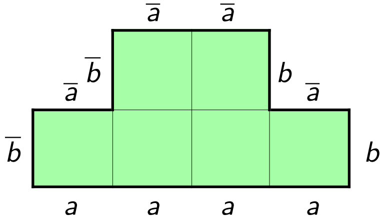

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A *tiling* of the plane by a polyomino p is a set T of non-overlapping translated copies of p that covers all the plane.

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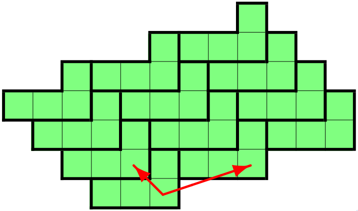
$$\exists u, v \in \mathbb{Z}^2, T = \{p_{iu+jv} | i, j \in \mathbb{Z}^2\}$$



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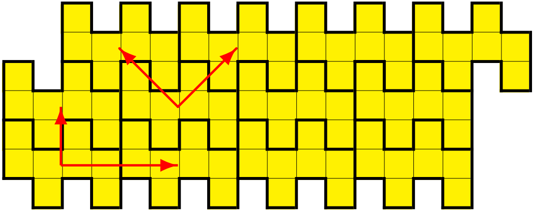

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Upper bound : ? ? ?

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1984 - Wijshof and Van Leeuwen

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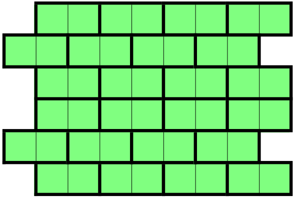
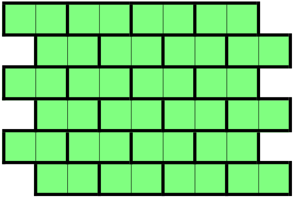
Irregular $\Longrightarrow$ Regular





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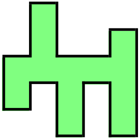

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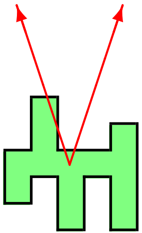

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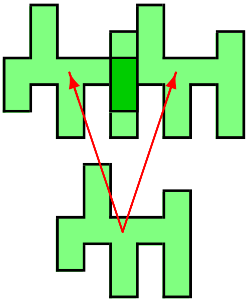

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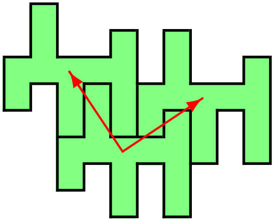

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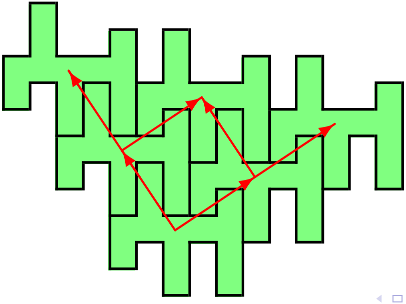

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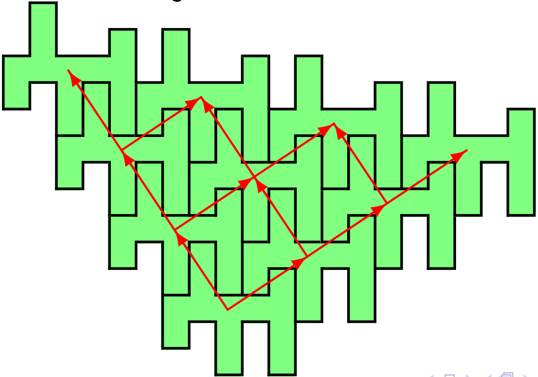

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Characterization : A polyomino p tiles the plane if and only if there exists $X, Y, Z \in \Sigma^*$ such that $w \equiv XYZ\widehat{X}\widehat{Y}\widehat{Z}$.

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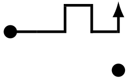

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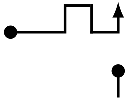

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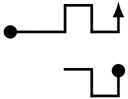

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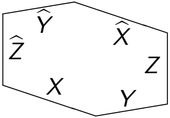
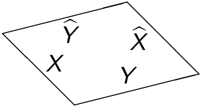


Pseudo-squares



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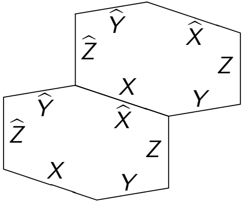

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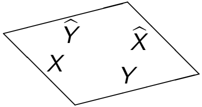

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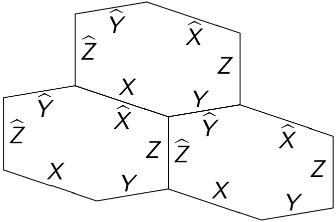

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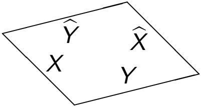

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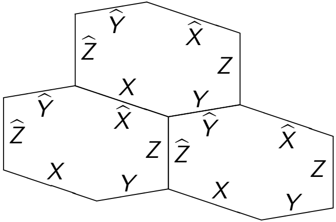

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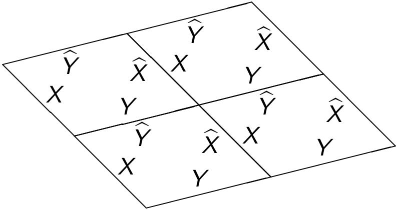

Gambini and Vuillon

2003 - Gambini and Vuillon

$\mathcal{O}(n^2)$ algorithm using Beauquier-Nivat's characterization.

Admissible factors

Definition

Let A be a factor of the word w coding a polyomino p. A is admissible if

- $w \equiv Ax\widehat{A}y$, *for* x, y *such that* $|x| = |y|$.
- A *is saturated, that is,* $x_0 \neq \overline{x_{k-1}}$ *and* $y_0 \neq \overline{y_{k-1}}$ *where* $k = |x| = |y|$.

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Lemma

Let w a word coding a polyomino p with Beauquier- Nivat's factorization $w \equiv XYZ\widehat{X}\widehat{Y}\widehat{Z}$. *Then,* X, Y *and* Z *are admissible.*

Admissible factors

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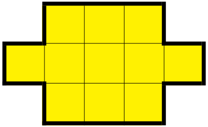

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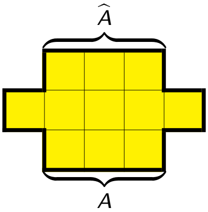
$$w \equiv \underbrace{a\,a\,a}_{A}\,\underbrace{b\,a\,b\,\bar{a}\,b}_{x}\,\underbrace{\bar{a}\,\bar{a}\,\bar{a}}_{\widehat{A}}\,\underbrace{\bar{b}\,\bar{a}\,\bar{b}\,a\,\bar{b}}_{y}$$



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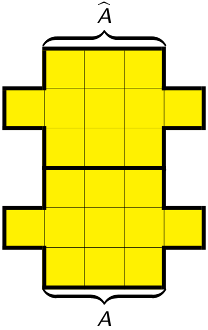

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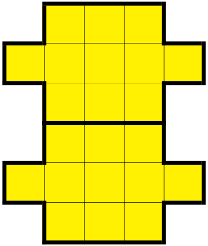

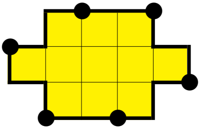

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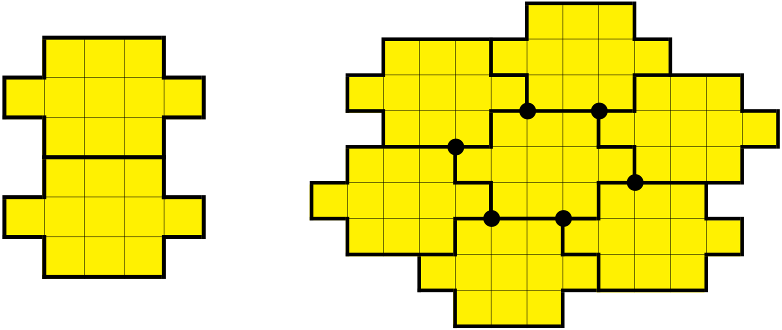

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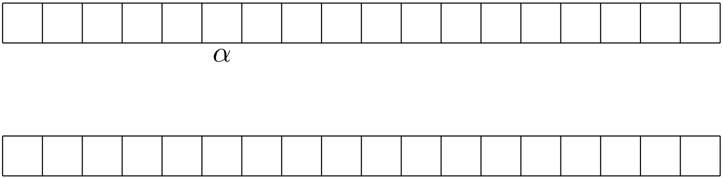

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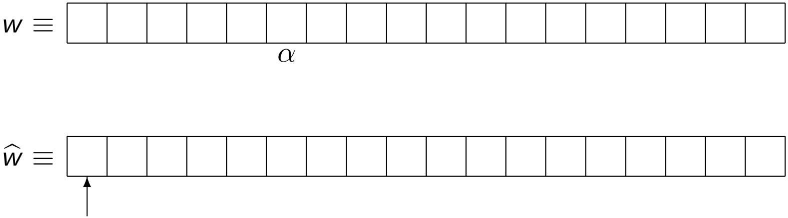

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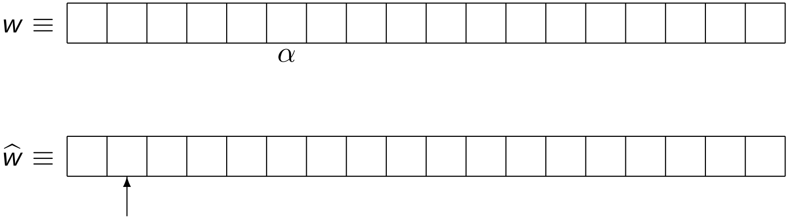

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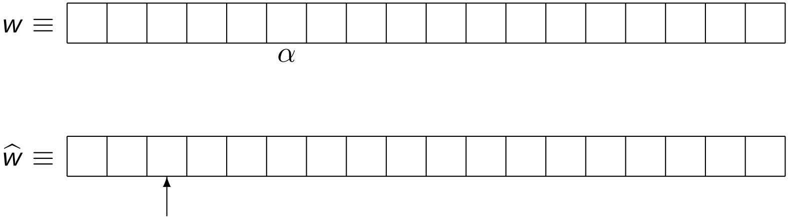

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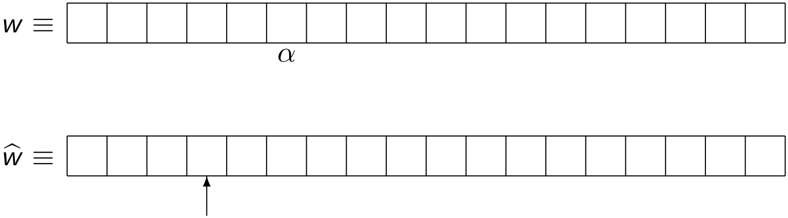

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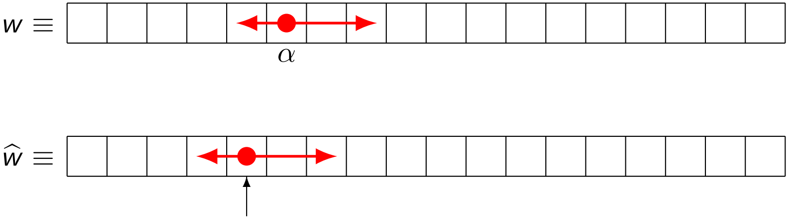

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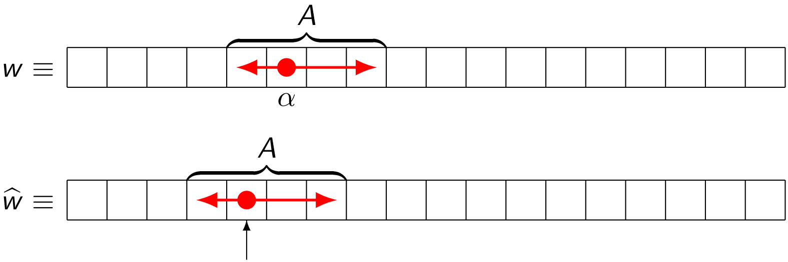

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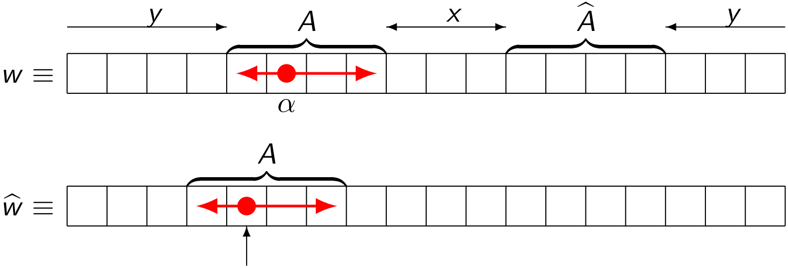

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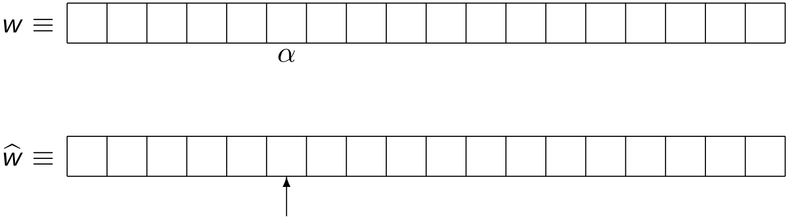

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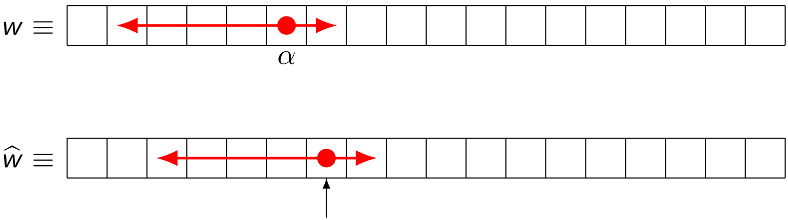

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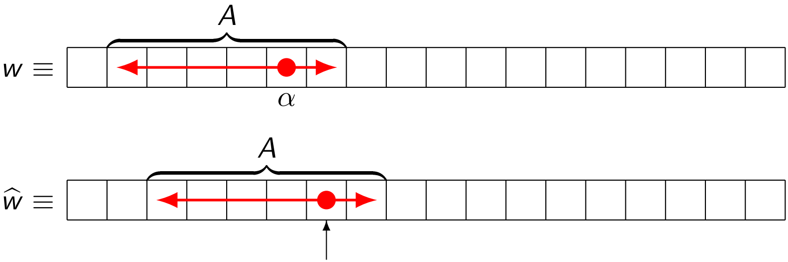

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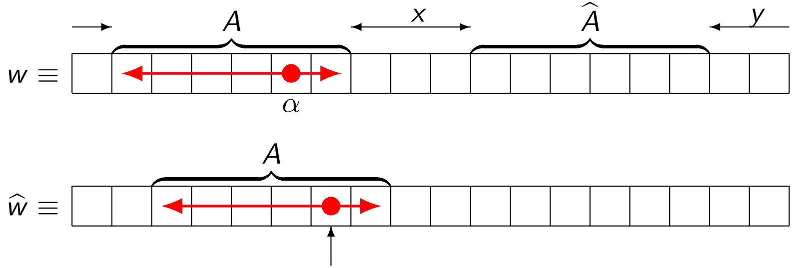

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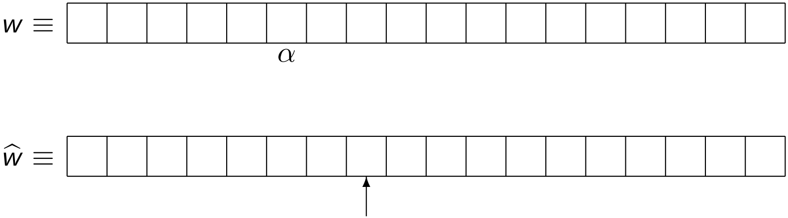

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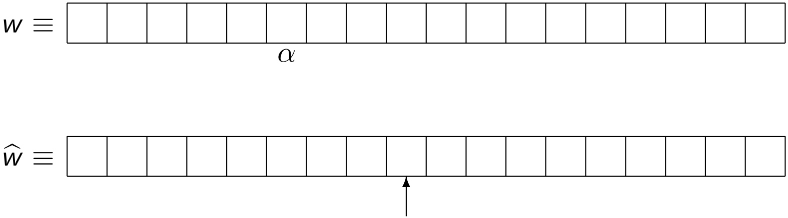

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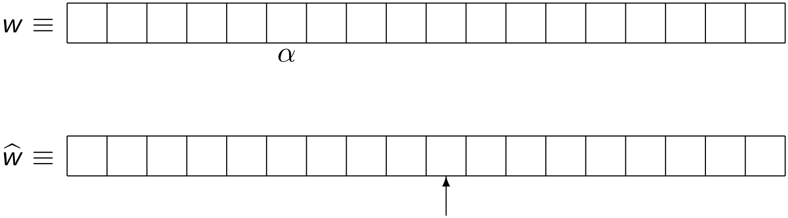

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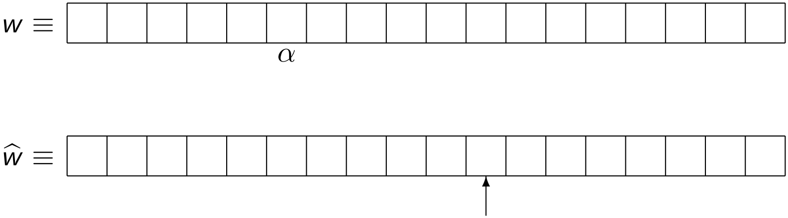

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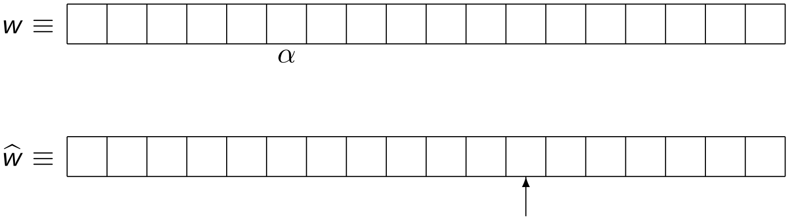

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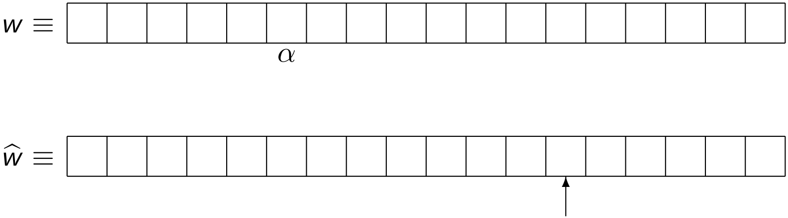

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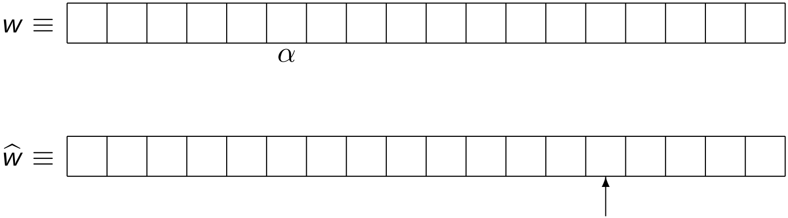

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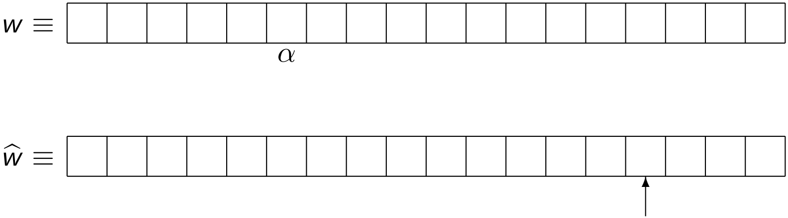

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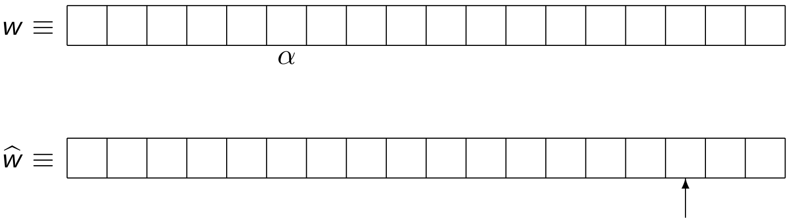

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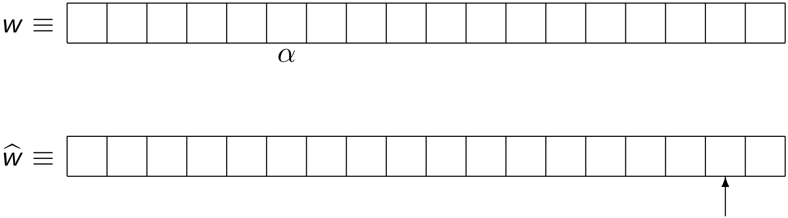

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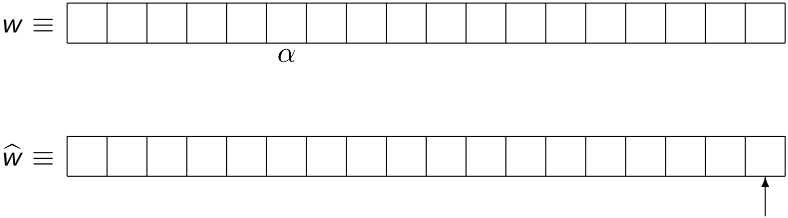

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Let w be the boundary of p. Determining if w codes a pseudo-square is decidable in linear time.

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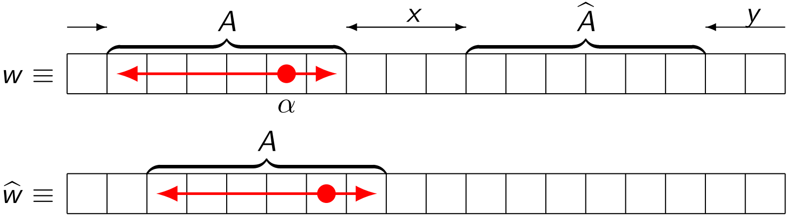

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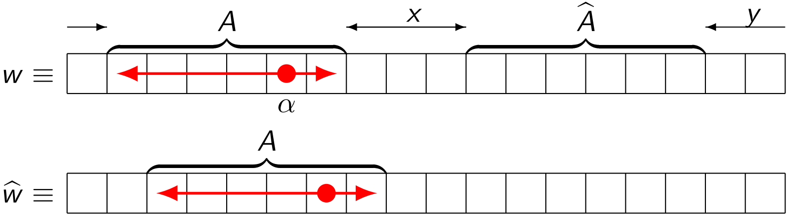

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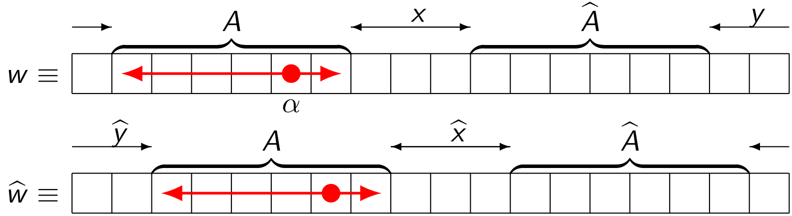

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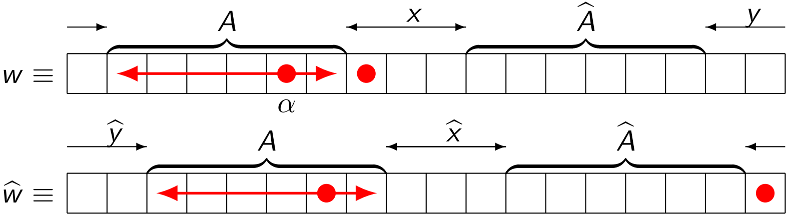

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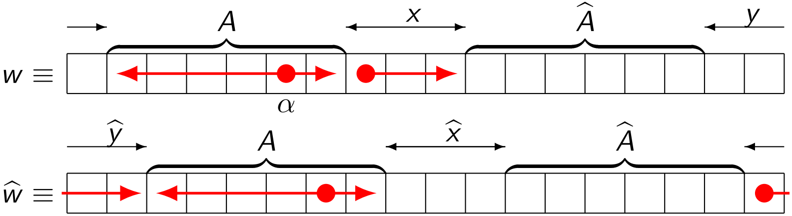

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Since $w \equiv A\,x\,\widehat{A}\,y$ then $\widehat{w} \equiv \widehat{y}\,A\,\widehat{x}\,\widehat{A}$.

Admissible factors

Lemma

Let $\mathcal{A}$ be the set of all admissible factors overlapping a position α in w and $\widehat{\mathcal{A}}$ be the set of their respective homologue factors. Then, there is at least one position in w that is not covered by any element of $\mathcal{A} \cup \widehat{\mathcal{A}}$.

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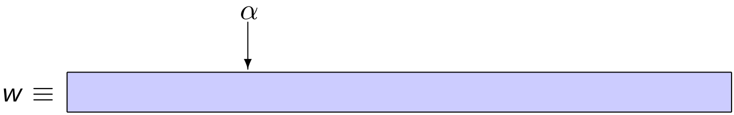

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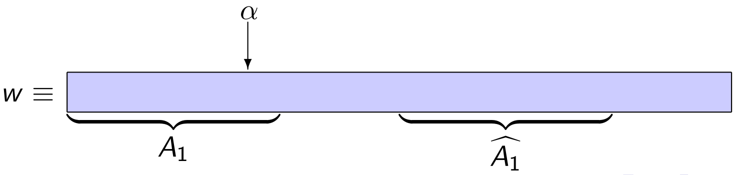

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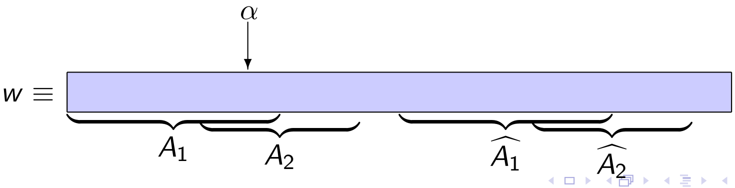

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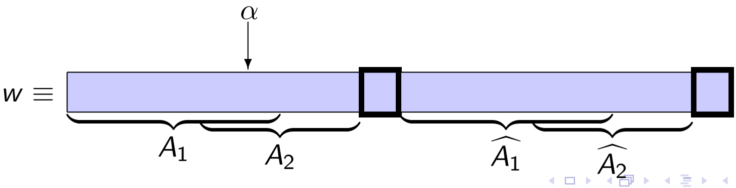

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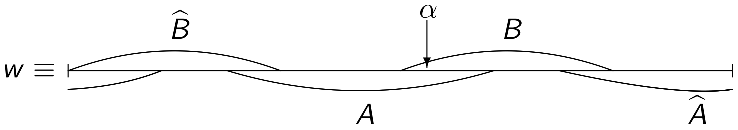

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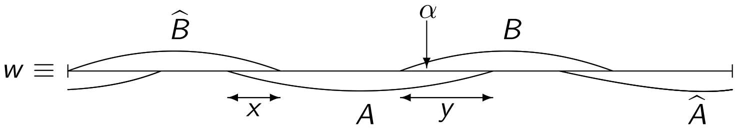

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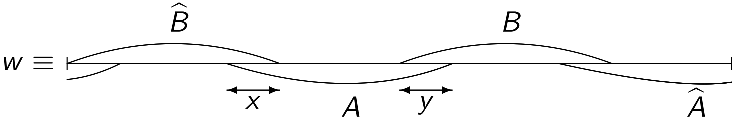

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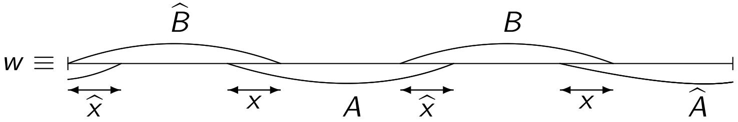

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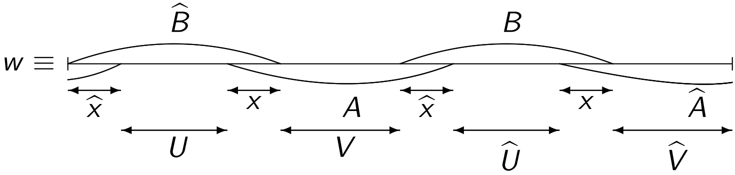

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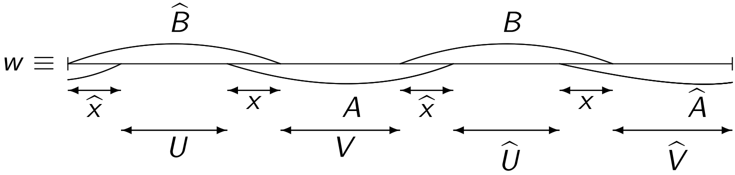

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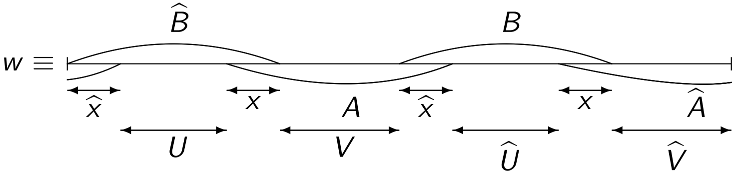

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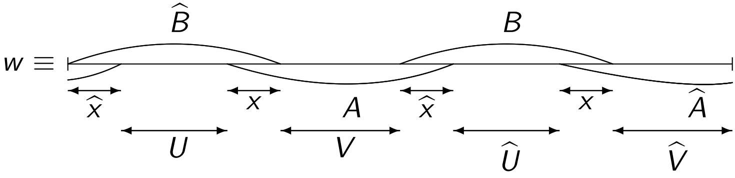

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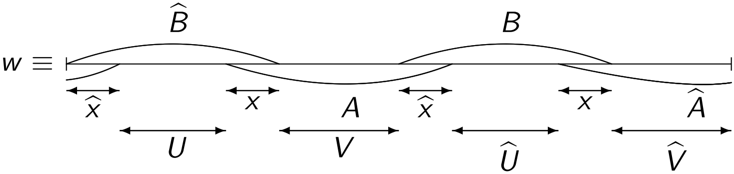

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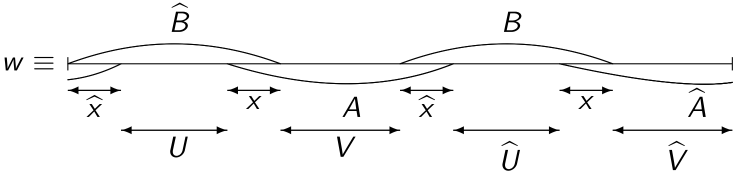

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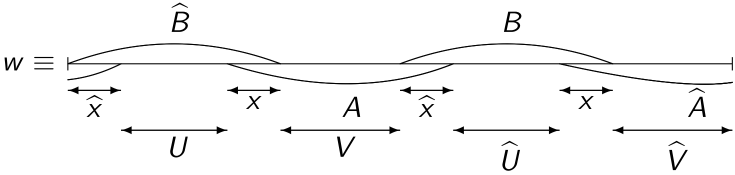

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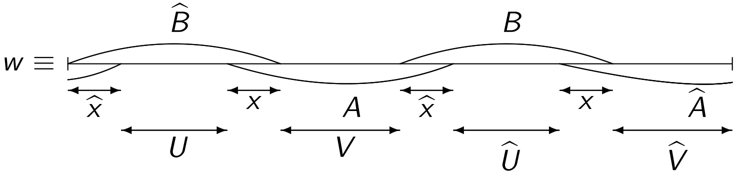

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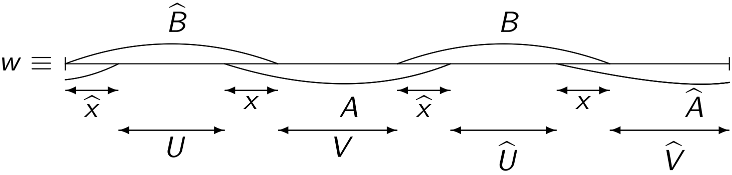

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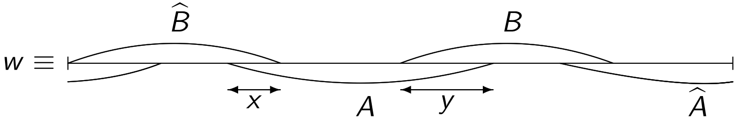

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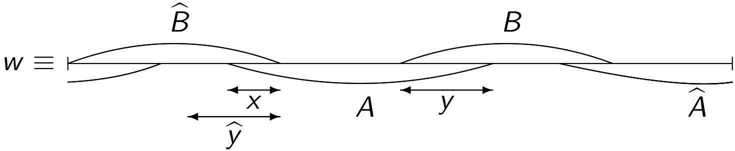

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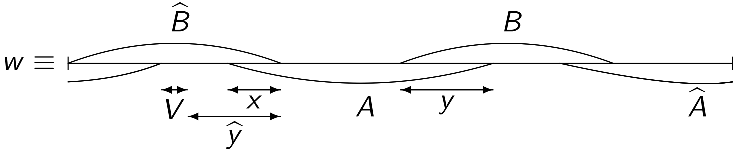

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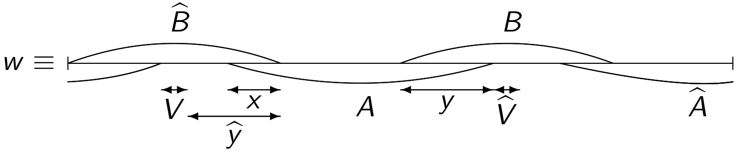

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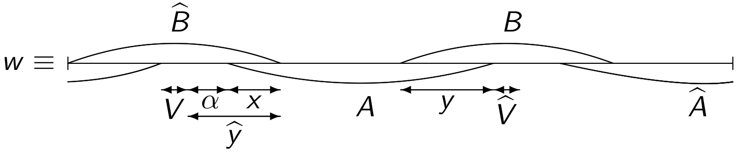

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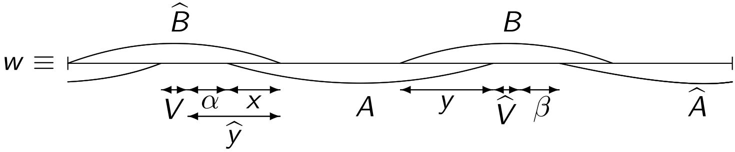

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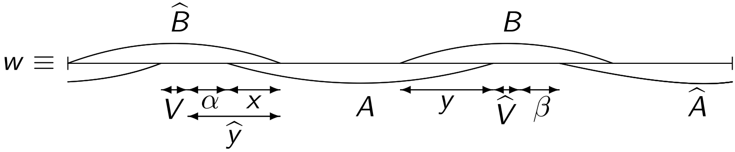

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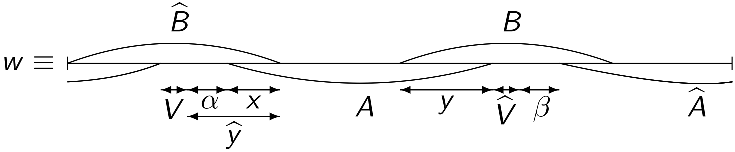

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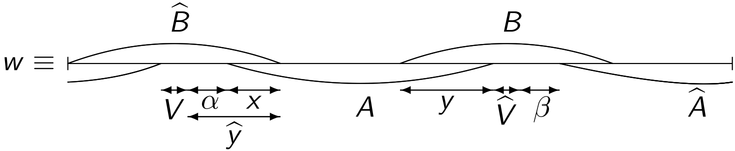

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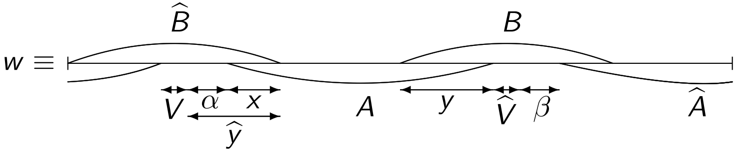

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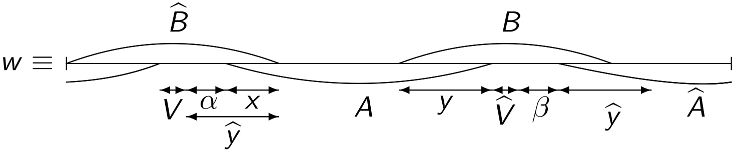

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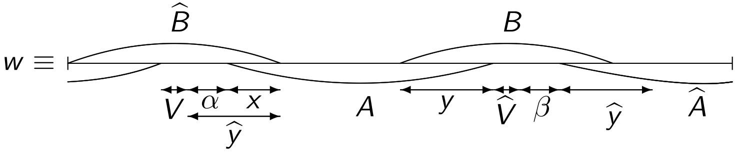

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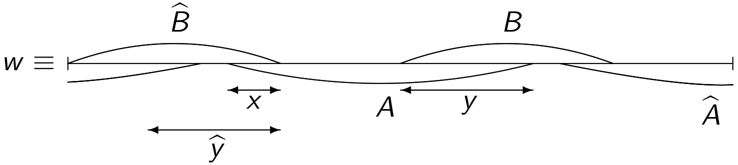

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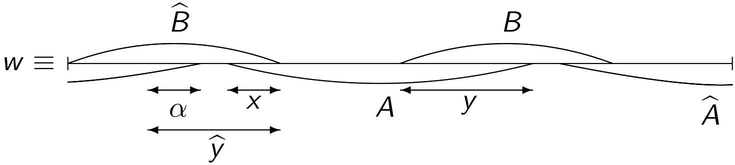

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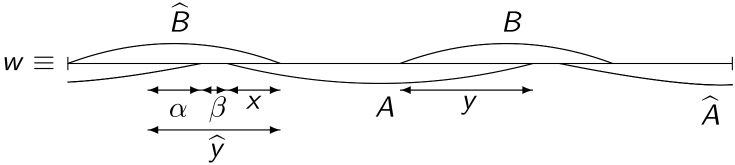

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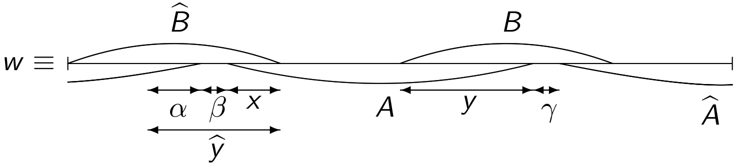

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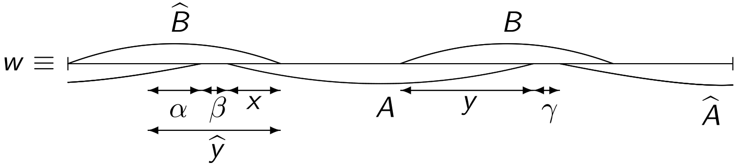

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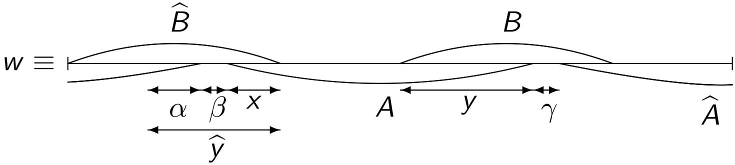

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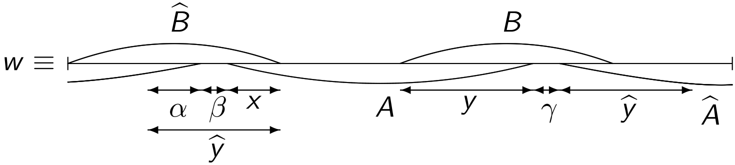

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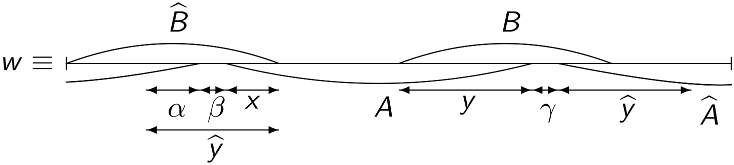

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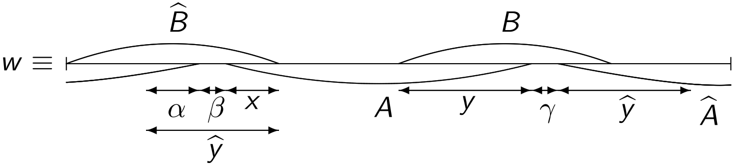

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k-square-free words

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A word w is k-square-free if

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Lemma

Let w be a k-square-free word coding a polyomino, and let α be a position in w. the number of admissible factors overlapping α in w is bounded by $4k + 2\log(n)$.

Detecting pseudo-hexagons

Theorem

Let w be a k-square-free word coding a polyomino, with $k \in \mathcal{O}(\sqrt{n})$. Determining if w codes a pseudo-hexagon is decidable in linear time.

Detecting pseudo-hexagons

Input : $w \in \Sigma^*$ coding a polyomino p.

Build L_1 the list of all admissible factors that overlap the position α.
$\beta :=$ (the position of the rightmost letter of w include in a factor of L_1) $+ 1$.
Build L_2 the list of all admissible factors that overlap the position β.
For all $X \in L_1$ **do**
  For all $Y \in L_2$ **do**
    If $w \equiv XYx\widehat{X}\widehat{Y}y$ **then**
      Compute i : the position of x in w.
      Compute j : the position of $\widehat{y}$ in $\widehat{w}$.
      If longest common extention$(w, \widehat{w}, i, j) = |x|$ **then**
        p is a *speudo-hexagon*.
      End if
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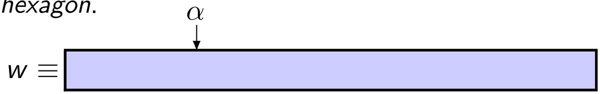

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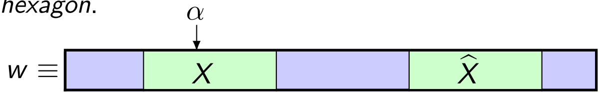

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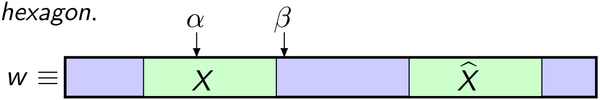

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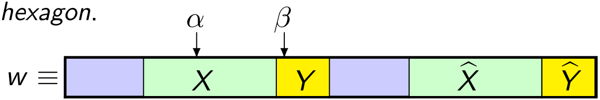

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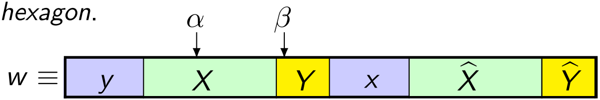

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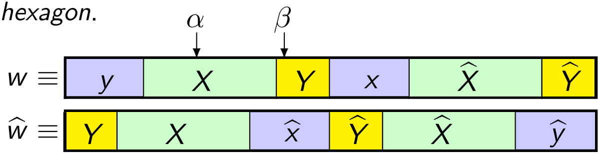

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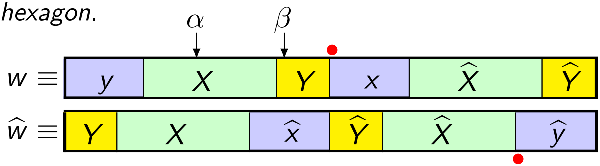

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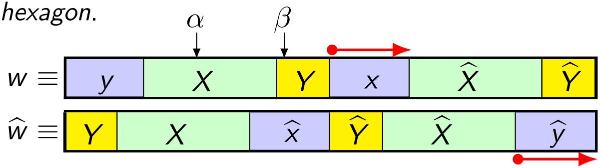

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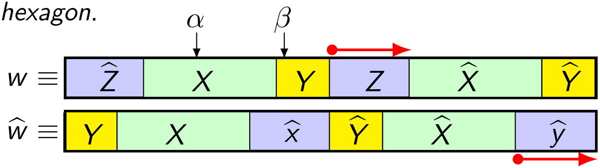

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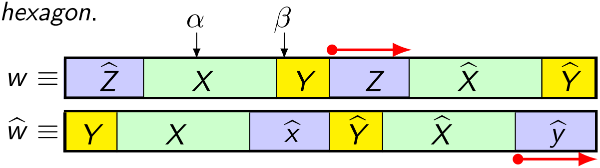
$$\mathcal{O}\left(n + (k + \log n)^2\right) = \mathcal{O}(n)$$



THANK YOU!